Math 307
Fall 2018
Exam 1 - Practice
9/20/18
Time Limit: 75 Minutes

Name (Print): _____

Problem	Points	Score
1	20	
2	30	
3	25	
4	20	
5	30	
6	20	
7	20	
Total:	165	

1. (20 points) Which of the following matrices are in reduced row-echelon form?

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & 8 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 2 & 3 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

2. (a) (5 points) $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} =$

(b) (5 points) $\begin{bmatrix} 2 & 4 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix} =$

(c) (5 points) If A is a 4×3 and B is a 3×4 matrix, what are the dimensions of AB and BA?

(d) (5 points) True of false: $\det(AB) = \det(A)\det(B)$

(e) (5 points) True of false: $\det(A + B) = \det(A) + \det(B)$

(f) (5 points) Prove or provide a counter example to the following statement: If A is invertible and B is invertible then $A + B$ is invertible.

3. (a) (15 points) Prove that A and B are row equivalent if and only if there is an invertible matrix C such that $CA = B$.

(b) (10 points) Compute

$$\det\left(\begin{bmatrix} 1 & 0 & 2 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 3 & 4 & 1 \\ -1 & -3 & -4 & -1 \end{bmatrix}\right)$$

4. (a) (10 points) Compute

$$\det\left(\begin{bmatrix} -1 & 0 & 0 & 2 \\ 1 & 1 & 2 & 1 \\ 1 & 0 & 0 & 1 \\ 2 & 0 & 4 & 0 \end{bmatrix}\right)$$

(b) (10 points) Suppose A is a 5×5 matrix such that $\det(A) = 2$. Let $E = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$.

What is $\det(EA) =$?

5. (a) (10 points) Show that $x^2+1, x-1$ are linearly independent.

(b) (10 points) Show that the vectors $x^2+1, x-1$ do not span P_2.

(c) (10 points) Show that $x^2+x+1,\ x^2-1,\ x+5,\ 4$ are linearly **dependent**.

6. (20 points) Suppose that

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Determine if A is invertible by computing $\det(A)$. If A *is* invertible, find A^{-1}.

7. (20 points) Let

$$A = \begin{bmatrix} 1 & 0 & -1 & 0 & 1 & 0 & 3 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 4 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 1 & 3 \end{bmatrix}$$

Find a basis for the null space and row space of A, the dimension of the null space of A, and the rank of A.