Math 307
Fall 2018
Final - Practice
12/4/18
Time Limit: 120 Minutes

Name (Print): _____

Problem	Points	Score
1	20	
2	15	
3	15	
4	20	
5	50	
6	25	
Total:	145	

1. (20 points) Answer true or false to the following statements:

   If V is a vector space of dimension n, then V is an $n \times n$ matrix.

   If 0 is an eigenvalue of A, then A is not invertible.

   If V is a vector space of dimension n, then any set of n vectors must be linearly independent.

   If $\det(A) \neq 0$ then A is invertible.

   If A is an $n \times n$ invertible matrix, then the rows of A are linerarly independent

   If v_1 and v_2 are linearly independent then there exists nonzero constants, c_1 and c_2, such that $c_1 v_1 + c_2 v_2 = 0$.

   If λ is an eigenvalue of A then $\det(\lambda I - A) = 0$.

   If V is a vector space of dimension n, then any collection of n vectors forms a basis for V.

   If A is an $n \times n$ matrix with n distinct eigenvalues, then A is diagonalizable.

   If A is an $n \times n$ matrix with n distinct eigenvalues, then A is invertible.

   If A is an $n \times n$ matrix that is invertible, then A is diagonalizable.

   If A is an $n \times n$ matrix that is diagonalizable, then A is invertible.

2. (15 points) Consider the matrix

$$\begin{bmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 3 \end{bmatrix}$$

Determine $\dim(E_1)$, $\dim(E_2)$ and $\dim(E_3)$.

3. Consider the homogeneous first order linear differential system

$$\begin{aligned} x' &= x - y + z \\ y' &= 2y + z \\ z' &= 2z \end{aligned}$$

where x, y, z are all functions of the variable t.

(a) (5 points) Write the system in the matrix form $Y' = AY + G$.

(b) (5 points) Is the matrix A diagonalizable? Explain your answer.

(c) (5 points) Find the general solution to the homogeneous system $Y' = AY$.

4. (20 points) Solve the initial value problem

$$Y' = \begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix} Y + \begin{bmatrix} x \\ 1 \end{bmatrix}, \qquad Y(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

5. Let's consider the non-homogeneous first order linear differential system

$$\begin{aligned} x' &= -4x - 3y + 3z \\ y' &= 3x + 2y - 3z + e^t \\ z' &= -3x - 3y + 2z \end{aligned}$$

where x, y, z are all functions of the variable t.

(a) (5 points) Write the system in the matrix form $Y' = AY + G$.

(b) (15 points) Find a diagonal matrix D and an invertible matrix P such that $D = P^{-1}AP$.

(c) (5 points) Find the general solution to the homogeneous system $Z' = DZ$.

(d) (5 points) Use the result of the previous question to find the general solution Y_H to the homogeneous system $Y' = AY$, and find a fundamental set, $Y_1,\ Y_2,\ Y_3$, of solutions to this system.

(e) (15 points) Using the matrix $M = [Y_1\ Y_2\ Y_3]$, compute a particular solution Y_p to the non-homogeneous system $Y' = AY + G$.

(f) (5 points) Use the result of the previous question to find the general solution to the non-homogeneous system $Y' = AY + G$.

6. Consider the following system:

$$y_1'' = y_1' + y_2' + e^x$$
$$y_2'' = y_1 + y_2 + \sin(x)$$

where y_1 and y_2 are functions of x.

(a) (5 points) Write this system as a system of first order liner differential equations.

(b) (5 points) Write the following differential equation as a system of first order linear equations:

$$y'' + 4y' + 3y = 0$$

(c) (10 points) Solve the system from part b).

(d) (5 points) Verify your solution by solving the original differential equation by using it's characteristic polynomial.