1. Find the leading term and use it to determine the long-term behavior of each polynomial function.

   (a) $f(x) = x^3 + 2x - 1$
   The leading term of $f(x)$ is x^3, so $f(x) \to \infty$ as $x \to \infty$ and $f(x) \to -\infty$ as $x \to -\infty$.

   (b) $f(x) = -x^2 + 4$
   The leading term of $f(x)$ is $-x^2$, so $f(x) \to -\infty$ as $x \to \pm\infty$.

   (c) $f(x) = -(x+2)^2(x-1)(x-3)^2$
   The leading term of $f(x)$ is the product of the leading terms of each factor:

   $$-1 \cdot (x)^2 \cdot x \cdot (x)^2 = -x^5$$

   So $f(x) \to -\infty$ as $x \to \infty$ and $f(x) \to \infty$ as $x \to -\infty$.

   (d) $f(x) = (x^2 + 2x - 1)^2(3x - 5)^4$
   The leading term of $f(x)$ is the product of the leading terms of each factor:

   $$(x^2)^2 \cdot (3x)^4 = 81x^8$$

   So $f(x) \to \infty$ as $x \to \pm\infty$.

2. Find all roots and their degrees. Describe the behavior of the graph at each root.

   (a) $g(x) = (x+1)(x-2)^2(x-4)$
   The function is already completely factored, so we read off the roots: $x = -1$ (degree 1), $x = 2$ (degree 2), and $x = 4$ (degree 1). The graph of $g(x)$ crosses the x-axis at $x = -1$ and $x = 4$. It only touches the x-axis at $x = 2$.

   (b) $g(x) = (2x-1)(x+6)^4$
   The function is already completely factored, so we read off the roots: $x = 1/2$ (degree 1) and $x = -6$ (degree 4). The graph of $g(x)$ crosses the x-axis at $x = 1/2$. It only touches the x-axis at $x = -6$.

   (c) $g(x) = (x^2 - 5x + 6)(x^2 - 16)$
   First we completely factor the function:

   $$g(x) = (x^2 - 5x + 6)(x^2 - 16) = (x-2)(x-3)(x+4)(x-4)$$

   Then we read off the roots: $x = -4$ (degree 1), $x = 2$ (degree 1), $x = 3$ (degree 1), $x = 4$ (degree 1). The graph of $g(x)$ crosses the x-axis at $x = -4, 2, 3, 4$.

(d) $g(x) = (x^2+1)(x^2-9)^2$
First we completely factor the function:

$$g(x) = (x^2+1)(x^2-9)^3 = (x^2+1)[(x+3)(x-3)]^2 = (x^2+1)(x+3)^2(x-3)^2$$

The roots of $g(x)$ are: $x=-3$ (degree 2) and $x=3$ (degree 2). Note that there are no roots from x^2+1. Thus the graph of $g(x)$ only touches the x-axis at $x=-3$ and at $x=3$.

3. Give the degree of each polynomial function. At most how many turning points does each graph have?

(a) $h(x) = x(x+7)(x-2)(x-5)$
This is a degree 4 polynomial, so it can have at most 3 turning points.

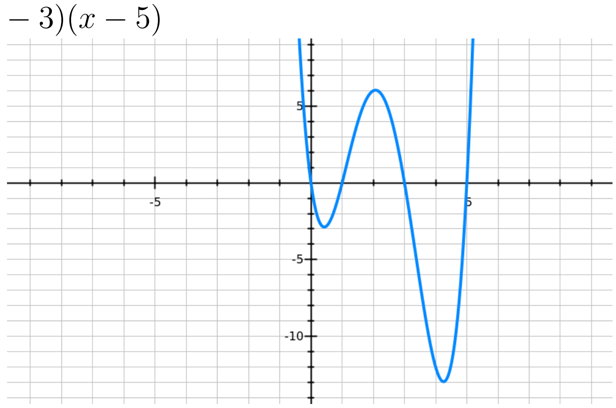
(b) $h(x) = (x-5)^2(x+3)^3$
This is a degree 5 polynomial, so it can have at most 4 turning points.

(c) $h(x) = (x^3+2x-1)^2$
This is a degree 6 polynomial, so it can have at most 5 turning points.

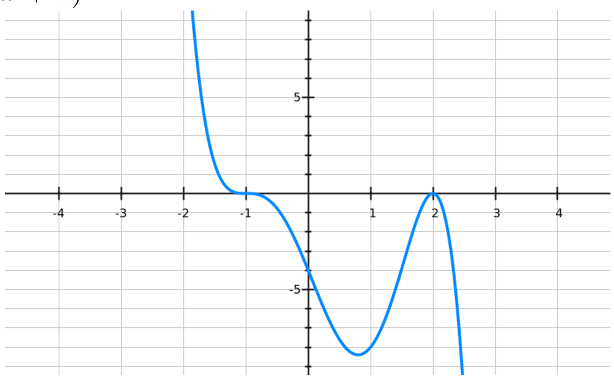
(d) $h(x) = x^2(3x+4)^2(x^2-3x+1)^3$
This is a degree 10 polynomial, so it can have at most 9 turning points.

4. Sketch the graph of each polynomial function. Label all roots with their degrees and mark all intercepts. The graph must be smooth and continuous.

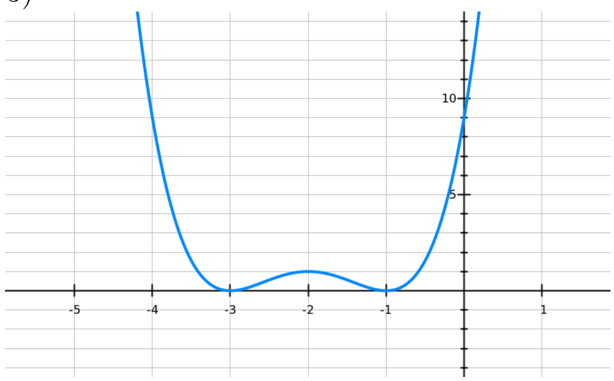
(a) $f(x) = x(x-1)(x-3)(x-5)$



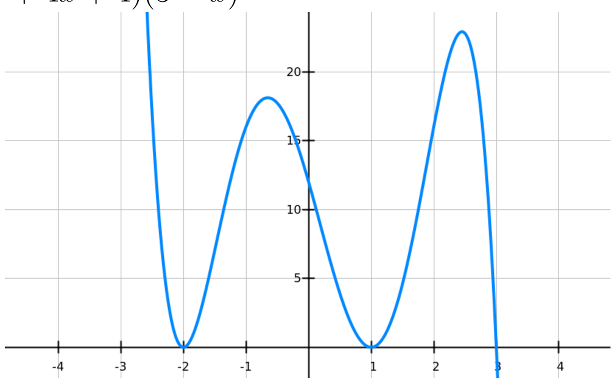
(b) $f(x) = -(x-2)^2(x+1)^3$



(c) $f(x) = (x^2+4x+3)^2$

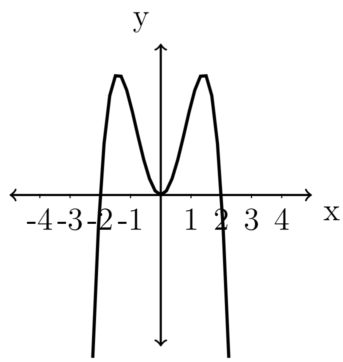


(d) $f(x) = (x-1)^2(x^2+4x+4)(3-x)$



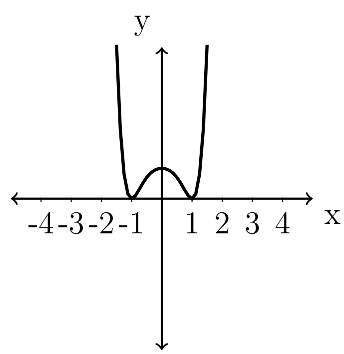
5. *Working backwards.* Find a possible polynomial function for each graph with the given degree. The y-axis is left intentionally without scale.

   (a) degree 4



   Possible function: $f(x) = -x^2(x+2)(x-2)$

   (b) degree 6



   Possible function: $f(x) = (x+1)^2(x-1)^2(x^2+1)$