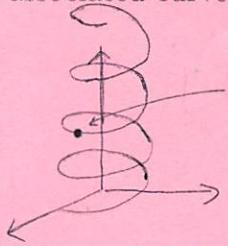


Name:

Section: 2 4 (circle one)

1. Let $r(t) = \cos(2\pi t)i + \sin(2\pi t)j + t^2k$ for $0 \leq t \leq 4$. Draw a reasonable sketch of the associated curve, and give the equation of the tangent line to the curve at $t = 2$.



$$\mathbf{r}(t) = -\sin(2\pi t)2\pi i + \cos(2\pi t)2\pi j + 2t k$$

$$\mathbf{r}'(2) = 0i + 2\pi j + 4k \leftarrow \text{This vector is parallel with}$$

$$\mathbf{r}(2) = 1i + 0j + 4k \quad \text{the tangent line.}$$

So, the equation of the tangent line is

$$x = 1, \quad y = 2\pi t, \quad z = 4 + 4t$$

2. For $r(t) = \cos(2\pi t)i + \sin(2\pi t)j + t^2$, find $\int_0^\pi r(t) dt$.
- $$\begin{aligned} &= \frac{\sin(2\pi t)}{2\pi} \Big|_0^\pi i + \frac{-\cos(2\pi t)}{2\pi} \Big|_0^\pi j + \frac{t^3}{3} \Big|_0^\pi k \\ &= \frac{\sin(2\pi^2)}{2\pi} i + \left(-\frac{\cos(2\pi^2)}{2\pi} + \frac{1}{2\pi} \right) j + \frac{\pi^3}{3} k \end{aligned}$$

3. Earth: A projectile is fired from the origin (the ground) at an initial speed of 100m/s and launch angle of $\pi/3$. Find the object's position after 10 seconds.

As was found in class, $\mathbf{r}(t) = V_0 \cos(\alpha) t i + \left[V_0 \sin(\alpha) t - \frac{gt^2}{2} \right] j$

Here, $V_0 = 100$, $g = 9.8$ and $\alpha = \pi/3$

So, the position after 10 seconds is

$$\begin{aligned} \mathbf{r}(10) &= 100 \cdot \frac{1}{2} \cdot 10 i + \left[100 \frac{\sqrt{3}}{2} \cdot 10 - \frac{9.8 \cdot 100}{2} \right] j \\ &= 500 i + [500\sqrt{3} - 490] j \end{aligned}$$

4. Planet Γ: A projectile is fired from the origin (the ground) at an initial speed of 100m/s and launch angle of $\pi/3$. Find the object's position after 10 seconds. (note: gravity on Planet Γ is $3\frac{\text{m}}{\text{s}^2}$).

$$\begin{aligned} \mathbf{r}(10) &= 100 \cdot \frac{1}{2} \cdot 10 \mathbf{i} + \left[100 \cdot \frac{\sqrt{3}}{2} \cdot 10 - \frac{3 \cdot 100}{2} \right] \mathbf{j} \\ &= 5,000 \mathbf{i} + [500\sqrt{3} - 150] \mathbf{j} \end{aligned}$$

5. For problems 3) and 4), determine where the objects hits the ground.

Since $x = v_0 \cos(\alpha) t$, we have $t = \frac{x}{v_0 \cos(\alpha)}$. This gives

$$y = v_0 \sin(\alpha) t - \frac{g t^2}{2}$$

$$y = \tan(\alpha) x - \frac{g}{2 v_0^2 \cos^2(\alpha)} x^2 = x \left(\tan(\alpha) - \frac{g}{2 v_0^2 \cos^2(\alpha)} x \right)$$

$$\text{so, if } y=0, \text{ then } x = \tan(\alpha) \cdot \frac{2 v_0^2 \cos^2(\alpha)}{g} = \frac{2 v_0^2 \sin(\alpha) \cos(\alpha)}{g}$$

$$\begin{aligned} \text{Earth: } x &= \frac{2,000\sqrt{3}/2 \cdot \frac{1}{2}}{9.8}, \quad T: x = \frac{2,000\sqrt{3}/2 \cdot \frac{1}{2}}{3} \\ &= \frac{5,000\sqrt{3}}{9.8} & &= \frac{5,000\sqrt{3}}{3} \end{aligned}$$

6. Determine the maximum height achieved by the object on Planet Γ.

$$\frac{dy}{dx} = \tan(\alpha) - \frac{g}{2 v_0^2 \cos^2(\alpha)} x, \text{ if } \frac{dy}{dx} = 0 \text{ we have } x = \tan(\alpha) \cdot \frac{v_0^2 \cos^2(\alpha)}{g}$$

$$= \frac{v_0^2 \sin(\alpha) \cos(\alpha)}{g}$$

For this x -value,

$$\begin{aligned} y &= \tan(\alpha) \left(\frac{v_0^2 \sin(\alpha) \cos(\alpha)}{g} \right) = \frac{g}{2 v_0^2 \cos^2(\alpha)} \cdot \left(\frac{v_0^2 \sin(\alpha) \cos(\alpha)}{g} \right)^2 \\ &= \frac{(v_0 \sin(\alpha))^2}{2g} \end{aligned}$$

Planet T: max height is
 $\frac{(100\sqrt{3}/2)^2}{6e} = 1,250$