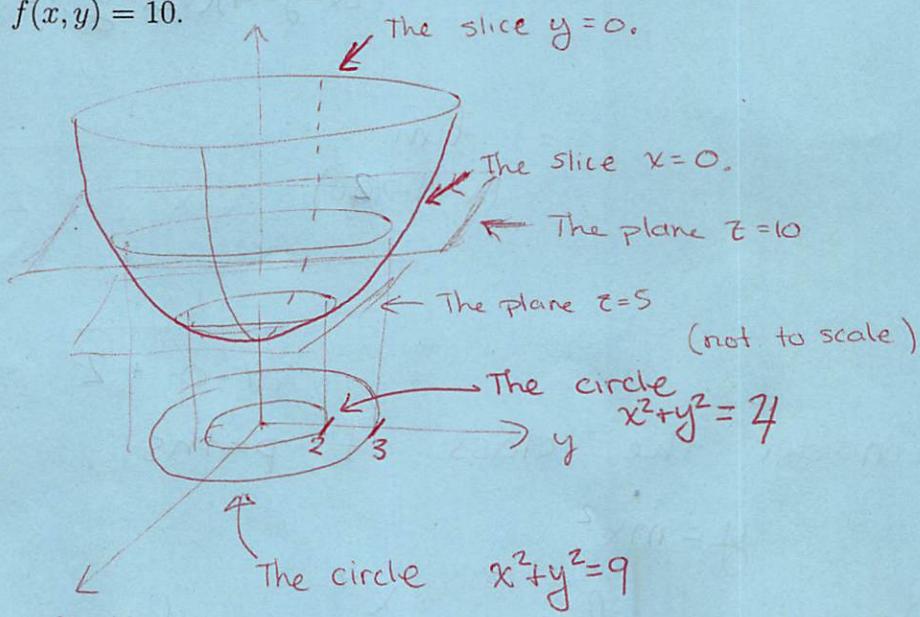


Name:

Schubert

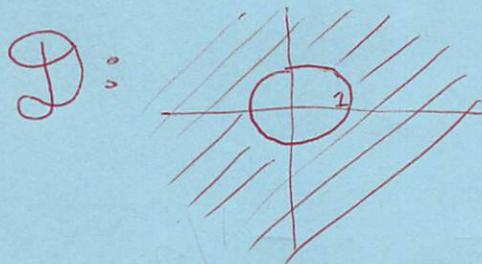
Section: 2 4 (circle one)

1. Sketch a graph of the function $z = f(x, y) = x^2 + y^2 + 1$ and in the xy -plane, sketch the level curves $f(x, y) = 5$ and $f(x, y) = 10$.

Sketch:

2. For the function $f(x, y) = \sqrt{x^2 + y^2 - 1}$, determine the domain, and range. Is the domain open, closed or neither? Is the domain bounded or unbounded?

$\mathcal{D}:$ it must be the case that $x^2 + y^2 - 1 \geq 0 \Leftrightarrow x^2 + y^2 \geq 1$,
 this second inequality can be interpreted as all circles
 w/ radius ≥ 1 . As such,



The boundary of \mathcal{D} is the circle of radius 1,
 and this is in \mathcal{D} , so
 \mathcal{D} is closed.

Clearly, this region is unbounded.

The Range is $[0, \infty)$ as $x^2 + y^2 - 1$ can be
 all values in $[0, \infty)$ and $\sqrt{}$ maps such #'s to $(0, \infty)$,

3. Determine the $\lim_{(x,y) \rightarrow (2,0)} \frac{\sqrt{2x-y} - 2}{2x-y-4} \cdot \frac{(\sqrt{2x-y} + 2)}{(\sqrt{2x-y} + 2)}$

$$= \lim_{(x,y) \rightarrow (2,0)} \frac{2x-y-4}{(2x-y-4)(\sqrt{2x-y} + 2)}$$

$$= \lim_{(x,y) \rightarrow (2,0)} \frac{1}{\sqrt{2x-y} + 2}$$

4. $\lim_{(x,y) \rightarrow (0,0)} \frac{x^4}{x^4 + y^2} = \frac{1}{\sqrt{4-0} + 2} = \frac{1}{4}$

Consider the class of paths,

$$y = mx^2$$

We have $\lim_{\substack{(x,y) \rightarrow (0,0) \\ y=mx^2}} \frac{x^4}{x^4 + y^2} = \lim_{x \rightarrow 0} \frac{x^4}{x^4 + (mx^2)^2}$

$$= \lim_{x \rightarrow 0} \frac{1}{1+m^2} = \frac{1}{1+m^2}$$

5. $\lim_{(x,y) \rightarrow (-1,0)} \frac{\sin(y)x + \sin(y)}{yx^2 - y}$

which depends on m,
 \therefore the limit does not exist!

$$= \lim_{(x,y) \rightarrow (-1,0)} \frac{\sin(y)}{y} \left(\frac{x+1}{x^2-1} \right)$$

$$= \lim_{(x,y) \rightarrow (-1,0)} \frac{\sin(y)}{y} \cdot \left(\frac{1}{x-1} \right)$$

Recall: $\lim_{\theta \rightarrow 0} \frac{\sin(\theta)}{\theta} = 1$

$$= 1 \cdot \frac{1}{-1-1} = -1/2$$