Math 242 Final Spring 2017

Name: ______________________________

☐ Section 1, Thursday 10:30-11:20, Sita Benedict

☐ Section 2, Thursday 1:30-2:20, Sita Benedict

☐ Section 3, Thursday 10:30-11:20, David Yuen

☐ Section 4, Thursday 12:00-12:50, David Yuen

☐ Section 5, Friday 11:30-12:20, Achilles Beros

☐ Section 6, Friday 2:30-3:20, Achilles Beros

☐ Section 7, Friday 8:30-9:20, Piper Harron

☐ Section 8, Friday 9:30-10:20, Piper Harron

☐ Section 9, Friday 10:30-11:20, Les Wilson

☐ Section 10, Friday 1:30-2:20, Les Wilson

Page	Points	Score
2	8	
3	6	
4	6	
5	8	
6	8	
7	6	
8	9	
9	4	
10	8	
11	12	
12	10	
13	7	
14	10	
15	8	
Total:	110	

- You may not use notes or calculators on the test.
- Please ask if anything seems confusing or ambiguous.
- You must show all your work and make clear what your final solution is (e.g. by drawing a box around it).
- The last two pages are a formula sheet. You are welcome to remove this from the exam.
- Good luck!

1. Circle either true or false. You do not need to justify your answer.

(a) (2 points) $\lim_{x\to+\infty} e^{3x} = +\infty$.

TRUE FALSE

(b) (2 points) $\lim_{x\to-\infty} e^{3x} = 0$.

TRUE FALSE

(c) (4 points) If f is a differentiable and one-to-one function, then

$$(f^{-1})'(x) = \frac{-1}{f'(x)},$$

provided the denominator is nonzero.

TRUE FALSE

2. For each of the following definite and indefinite integrals, evaluate it or show that it diverges.

   (a) (6 points) $\int_0^1 2xe^x\,dx$

(b) (6 points) $\int \frac{x^2}{1+x^2} dx$

(c) (8 points) $\displaystyle\int_{-1}^{1} \frac{3x-2}{x^2+x-12}dx$

(d) (8 points) $\int_1^\infty \frac{\ln(x)}{x^2} dx$

3. (6 points) Find the derivative of $g(x) = \left(\sin^{-1}(x)\right)^x$.

4. For each, determine if the given limit exists and find it if it does (you must justify any use of l'Hôpital's rule).

   (a) (4 points) $\lim_{x \to 0^+} \sqrt{x} \ln\left(x^3\right)$

   (b) (5 points) $\lim_{x \to +\infty} x^{3/x}$

5. (4 points) Find an upper bound for the error (using the relevant formula from the formula sheet) when one uses the Trapezoidal rule with $n = 4$ to estimate $\int_{-1}^{1} e^{x^2}\,dx$. (**Note: you do not need to find the approximation, only an upper bound for the error**).

6. Circle either true or false. You do not need to justify your answer.

(a) (4 points) The series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ converges but not absolutely. In other words, it converges conditionally.

TRUE FALSE

(b) (4 points) The sum of the series $\sum_{n=2}^{\infty} \frac{2}{5^n}$ is $\frac{1}{10}$.

TRUE FALSE

7. For each of the following series decide if it converges or diverges and explain why by explicitly stating which test(s) are used in your solution.

   (a) (6 points) $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{n^2}$

   (b) (6 points) $\displaystyle\sum_{n=1}^{\infty} \frac{\tan^{-1}(n)}{n^2}$

8. Consider the power series $\sum_{n=1}^{\infty} \frac{(x-4)^n}{3^n \sqrt{n}}$.

(a) (2 points) What is the center of the power series?

(b) (6 points) What is radius of convergence of the power series?

(c) (2 points) Does the power series converge absolutely at $x = 2$? Justify your answer.

9. (7 points) Compute the Taylor polynomial of order 2 for the function $f(x) = \sqrt{x+4}$ centered at $x = 0$.

10. (10 points) Find the general solution of the following differential equation

$$y' + \frac{1}{x}y = \frac{\sin^3(x)}{x}, \quad x > 0.$$

11. (8 points) Solve the initial value problem

$$y'' - 6y' + 8y = 0 \qquad y(0) = 0, \qquad y'(0) = 2$$

Formula sheet

- Derivatives of inverse trigonometric functions.

$$\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\cos^{-1}(x) = -\frac{1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2} \qquad \frac{d}{dx}\cot^{-1}(x) = -\frac{1}{1+x^2}$$
$$\frac{d}{dx}\sec^{-1}(x) = \frac{1}{|x|\sqrt{x^2-1}} \qquad \frac{d}{dx}\csc^{-1}(x) = -\frac{1}{|x|\sqrt{x^2-1}}$$

- Trigonometric identities.

$$\begin{aligned}
\sin^2 x + \cos^2 x &= 1 \\
1 + \tan^2 x &= \sec^2 x \\
1 + \cot^2 x &= \csc^2 x \\
\sin^2 x &= \frac{1}{2}(1 - \cos(2x)) \\
\cos^2 x &= \frac{1}{2}(1 + \cos(2x)) \\
\sin x \cos x &= \frac{1}{2}\sin(2x) \\
\sin x \sin y &= \frac{1}{2}\cos(x-y) - \frac{1}{2}\cos(x+y) \\
\cos x \cos y &= \frac{1}{2}\cos(x-y) + \frac{1}{2}\cos(x+y) \\
\sin x \cos y &= \frac{1}{2}\sin(x-y) + \frac{1}{2}\sin(x+y)
\end{aligned}$$

- Integrals of trigonometric functions.

$$\begin{aligned}
\int \tan x \; dx &= \ln|\sec x| + C \\
\int \cot x \; dx &= \ln|\sin x| + C \\
\int \sec x \; dx &= \ln|\sec x + \tan x| + C \\
\int \csc x \; dx &= -\ln|\csc x + \cot x| + C
\end{aligned}$$

- Trapezoidal Rule and Simpson's Rule.

$$T = \frac{\Delta x}{2}\left(y_0 + 2y_1 + 2y_2 + \ldots + 2y_{n-1} + y_n\right)$$
$$S = \frac{\Delta x}{3}\left(y_0 + 4y_1 + 2y_2 + 4y_3 + \ldots + 2y_{n-2} + 4y_{n-1} + y_n\right)$$

- Error estimates for Trapezoidal Rule and Simpson's Rule.

$$|E_T| \leq \frac{M(b-a)^3}{12n^2}, \quad \text{where } |f''(x)| \leq M \text{ for all } x \text{ in } [a,b]$$
$$|E_S| \leq \frac{M(b-a)^5}{180n^4}, \quad \text{where } |f^{(4)}(x)| \leq M \text{ for all } x \text{ in } [a,b]$$

- Famous Maclaurin series.

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \qquad (R = \infty)$$
$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} \qquad (R = \infty)$$
$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} \qquad (R = \infty)$$
$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^n}{n} \qquad (R = 1)$$
$$\tan^{-1} x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1} \qquad (R = 1)$$

- Error estimate for approximations by Taylor polynomials.

$$|R_n(x)| \leq \frac{M|x-a|^{n+1}}{(n+1)!},$$

where $|f^{(n+1)}(t)| \leq M$ for all t between a and x.