Name: Section: 5 6 9 10

1. Determine if each of the following series converges or diverges. You may use the techniques of geometric series, telescoping series, p-series, divergence test, integral test, comparison test, limit comparison test.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n + \ln(n)}{3^n + n}$ behaves like $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{3^n}$

   $$\frac{2^n + \ln(n)}{3^n + n} \leq \frac{2^n + 2^n}{3^n} = 2 \cdot \left(\frac{2}{3}\right)^n$$

   Since $\displaystyle\sum_{n=1}^{\infty} 2 \cdot \left(\frac{2}{3}\right)^n$ converges (geometric $|r| = \left|\frac{2}{3}\right| < 1$)

   $\Longrightarrow \displaystyle\sum_{n=1}^{\infty} \frac{2^n + \ln(n)}{3^n + n}$ converges by comparison

   Alternatively, limit compare with $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{3^n}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n} + 2n}$ try AST.

   Let $b_n = \dfrac{1}{\sqrt{n} + 2n}$

   (i) b_n's are positive

   (ii) $b_{n+1} \leq b_n$ since denominator gets larger

   (iii) $\displaystyle\lim_{n\to\infty} \frac{1}{\sqrt{n} + 2n} = 0$ Type $\frac{1}{\infty}$

   Series converges