

Name:

Section: 5 6 9 10

Pick TWO of the following three series and determine if they converge absolutely, converge conditionally, or diverge. Clearly state which tests you are using and justify your work.

$$(a) \sum_{n=1}^{\infty} \frac{n}{n+2}$$

$$(b) \sum_{n=1}^{\infty} \frac{e^n}{n!}$$

$$(c) \sum_{n=1}^{\infty} \frac{(-1)^n}{n+1}$$

(a) Divergence test: $\lim_{n \rightarrow \infty} \frac{n}{n+2} = \frac{1}{1} = 1 \neq 0$
series diverges

(b) ~~Series~~ Try Ratio test:

$$\left| \frac{a_{n+1}}{a_n} \right| = \left| \frac{e^{n+1}}{(n+1)!} \cdot \frac{n!}{e^n} \right| = \frac{e}{n+1} \rightarrow 0 < 1$$

Series converges absolutely.

(c) $\sum_{n=1}^{\infty} \left| \frac{(-1)^n}{n+1} \right| = \sum_{n=1}^{\infty} \frac{1}{n+1}$ comparison test:

$$\frac{1}{n+1} \geq \frac{1}{n+2} = \frac{1}{2} \cdot \frac{1}{n}. \quad \text{Since } \sum_{n=1}^{\infty} \frac{1}{2} \cdot \frac{1}{n} \text{ diverges}$$

(p-series $p=1$), then $\sum_{n=1}^{\infty} \frac{1}{n+1}$ doesn't converge.

Next, try AST: $b_n = \frac{1}{n+1}$

yes, b_n 's are positive & decreasing, and

$$\lim_{n \rightarrow \infty} b_n = \lim_{n \rightarrow \infty} \frac{1}{n+1} = 0$$

\therefore series converges conditionally.