Name: Solutions. Section: 5 6 9 10

1. Differentiate the following functions. You do not need to simplify :)

(a) $f(x) = \tan(\ln x) + \ln(2x^3 - 9x)$.

$$f'(x) = \sec^2(\ln x) \cdot \frac{1}{x} + \frac{1}{2x^3 - 9x} \cdot (6x^2 - 9)$$

(b) $s(x) = e^{2x-1} + e^{x \ln 3}$.

$$s'(x) = e^{2x-1} \cdot (2) + e^{x \cdot \ln 3} \cdot (\ln 3)$$

(c) $g(x) = (\tan x)^{\cot x}$ use logarithmic differentiation

Put $y = (\tan x)^{\cot x}$

$$\Rightarrow \ln y = \cot x \cdot \ln(\tan x)$$

$$\Rightarrow \frac{1}{y} \cdot \frac{dy}{dx} = -\csc^2 x \cdot \ln(\tan x) + \cot x \cdot \frac{1}{\tan x} \cdot \sec^2 x$$

$$\Rightarrow \frac{dy}{dx} = y\left(-\csc^2 x \cdot \ln(\tan x) + \csc^2 x\right)$$

$$\therefore g'(x) = (\tan x)^{\cot x} \cdot \left(\csc^2 x - \csc^2 x \cdot \ln(\tan x)\right)$$

$$= (\tan x)^{\cot x} \cdot \csc^2 x \cdot \left(1 - \ln(\tan x)\right)$$

2. Evaluate the integrals

(a) $\int \frac{3x^3}{x^4+3}dx$ $= 3 \cdot \int \frac{x^3\,dx}{x^4+3}$

$u = x^4 + 3$

$du = 4x^3\,dx$

$\frac{1}{4}du = x^3\,dx$

$= 3 \cdot \int \frac{\frac{1}{4}du}{u}$

$= \frac{3}{4} \cdot \int \frac{du}{u}$

$= \frac{3}{4} \cdot \ln|u| + C$

$= \frac{3}{4} \cdot \ln|x^4+3| + C$

(b) $\int \sin x e^{\cos x} dx$ $= \int e^{\cos x} \cdot (\sin x\, dx)$

$u = \cos x$

$du = -\sin x\, dx$

$-du = \sin x\, dx$

$= \int e^u \cdot (-du)$

$= -\int e^u\, du$

$= -e^u + C$

$= -e^{\cos x} + C$