

Name:

Solutions

Section: 5 6 9 10

1. Find the following values.

$$(a) \log_3\left(\frac{1}{27}\right) = x \iff 3^x = \frac{1}{27} \iff x = -3$$

$$(b) \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = x \iff \cos x = \frac{1}{\sqrt{2}} \iff x = \pi/4$$

$$(c) \tan^{-1}(-\sqrt{3}) = x \iff \tan x = -\sqrt{3} \iff x = -\pi/3$$

2. Differentiate the following functions.

$$(a) f(x) = \ln(\sin^{-1}(x)) + \sec^{-1}(\sqrt{x}).$$

$$f'(x) = \frac{1}{\sin^{-1}(x)} \cdot \frac{1}{\sqrt{1-x^2}} \cdot \frac{1}{(\sqrt{x}) \cdot \sqrt{(\sqrt{x})^2 - 1}} \cdot \frac{1}{2} x^{-1/2}$$

$$(b) y = \arctan(2e^x).$$

$$y' = \frac{1}{1 + (2e^x)^2} \cdot 2e^x.$$

3. Suppose you have a 100mg sample of cesium-137. The half-life of cesium-137 is 30 years. Find the mass of the sample after 72 years.

$$m_0 = 100$$

$$m(30) = 50$$

$$m = 100e^{k \cdot t}$$

$$50 = 100e^{k \cdot 30}$$

$$\frac{1}{2} = e^{k \cdot 30}$$

$$\ln \frac{1}{2} = k \cdot 30$$

$$k = \frac{1}{30} \ln \frac{1}{2}$$

$$m(72) = 100e^{\frac{1}{30} \ln \frac{1}{2} \cdot 72} \text{ mg.}$$

4. Evaluate the following integrals.

(a) Find $\int \frac{2x}{1+x^4} dx$. *Hint: let $u = x^2$.*

$$\begin{aligned}
 u &= x^2 \\
 du &= 2x dx \\
 &= \int \frac{du}{1+u^2} \\
 &= \arctan(u) + C \\
 &= \arctan(x^2) + C.
 \end{aligned}$$

(b) Evaluate $\int \frac{1}{\sqrt{9-4x^2}} dx$. *Hint: factor 3 out of the denominator and let $u = \frac{2x}{3}$.*

$$\begin{aligned}
 &= \int \frac{1}{3\sqrt{1-\left(\frac{2x}{3}\right)^2}} dx = \int \frac{1}{\sqrt{1-\left(\frac{2x}{3}\right)^2}} \left(\frac{dx}{3}\right) \\
 u &= \frac{2x}{3} \\
 du &= \frac{2}{3} dx \\
 \frac{1}{2} du &= \frac{1}{3} dx \\
 &= \int \frac{1}{\sqrt{1-u^2}} \left(\frac{du}{2}\right) \\
 &= \frac{1}{2} \sin^{-1}(u) + C \\
 &= \frac{1}{2} \sin^{-1}\left(\frac{2x}{3}\right) + C
 \end{aligned}$$