Name: Section: 5 6 9 10

1. Use integration by parts to evaluate the following integrals.

(a) $\int xe^{2x} dx$

$u = x \qquad dv = e^{2x}\,dx$

$du = dx \qquad v = \frac{1}{2}e^{2x}\,dx$

$= \frac{1}{2}xe^{2x} - \int \frac{1}{2}e^{2x}\,dx$

$= \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + C$

(b) $\int x\ln(4x)dx$

$u = \ln(4x) \qquad dv = x\,dx$

$du = \frac{1}{x}\,dx \qquad v = \frac{1}{2}x^2$

$= \ln(4x)\cdot\frac{1}{2}x^2 - \int \frac{1}{2}x^2\cdot\frac{1}{x}\,dx$

$= \frac{1}{2}x\ln(4x) - \frac{1}{2}\int x\,dx$

$= \frac{1}{2}x\ln(4x) - \frac{1}{4}x^2 + C$

2. Use the methods we discussed in lecture to find the following limits.

(a) $\displaystyle\lim_{x\to\frac{\pi}{2}^+} \frac{\cos x}{1-\sin x}$ Type $\frac{0}{0}$

$$\overset{H}{=} \lim_{x\to\frac{\pi}{2}^+} \frac{-\sin x}{-\cos x} = \lim_{x\to\frac{\pi}{2}^+} \tan x = -\infty$$

(b) $\displaystyle\lim_{x\to 0^+} x\ln(\tan x)$ Type $0\cdot(-\infty)$

$$= \lim_{x\to 0^+} \frac{\ln(\tan x)}{1/x} \quad \text{Type } \frac{-\infty}{\infty}$$

$$\overset{H}{=} \lim_{x\to 0^+} \frac{\frac{1}{\tan x}\cdot\sec^2 x}{-x^{-2}}$$

$$= \lim_{x\to 0^+} -\frac{(x\sec x)^2}{\tan x} \quad \text{Type } \frac{0}{0}$$

$$\overset{H}{=} \lim_{x\to 0^+} \frac{-2(x\sec x)[\sec x + x\sec x\cdot\tan x]}{\sec^2 x}$$

$$= 0$$

(c) $\displaystyle\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^{3x}$ Type 1^∞

Set $\displaystyle L = \lim_{x\to\infty}\left(1+\frac{2}{x}\right)^{3x}$

$$\Rightarrow \ln L = \lim_{x\to\infty} 3x\cdot\ln\left(1+\frac{2}{x}\right) \quad \text{Type } \infty\cdot 0$$

$$= 3\cdot\lim_{x\to\infty}\frac{\ln\left(1+\frac{2}{x}\right)}{1/x} \quad \text{Type } \frac{0}{0}$$

$$\overset{H}{=} 3\cdot\lim_{x\to\infty}\frac{\frac{x}{x+2}\cdot(-2\cancel{x^{-2}})}{\cancel{-x^{-2}}}$$

$$= 6\cdot\lim_{x\to\infty}\frac{x}{x+2}$$

$$= 6\cdot 1$$

$$= 6$$

$$\therefore L = e^6$$