Name: Section: 5 6 9 10

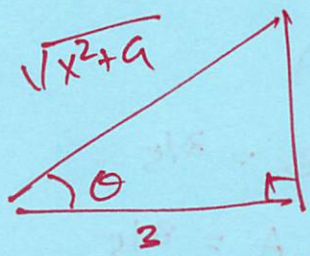
1. Use a trigonometric substitution to evaluate $\int \frac{dx}{x^2\sqrt{9+x^2}}$

Pattern $\sqrt{9+x^2}$

$x = 3\tan\theta$

- $x^2 = 9\tan^2\theta$
- $dx = 3\sec^2\theta\, d\theta$
- $\sqrt{9+x^2} = \sqrt{9+9\tan^2\theta}$

$= 3\sec\theta$



$\tan\theta = \frac{x}{3} = \frac{opp}{\cancel{hyp}\, adj}$

$\csc\theta = \frac{hyp}{opp} = \frac{\sqrt{x^2+9}}{x}$

$$= \int \frac{3\sec^2\theta\, d\theta}{9\tan^2\theta \cdot 3\sec\theta}$$

$$= \frac{1}{9}\int \frac{\sec\theta}{\tan^2\theta}\, d\theta$$

$$= \frac{1}{9}\int \frac{\cos^2\theta}{\sin^2\theta} \cdot \frac{1}{\cos\theta}\, d\theta$$

$$= \frac{1}{9}\int \frac{\cos\theta}{\sin^2\theta}\, d\theta$$

$u = \sin\theta$
$du = \cos\theta\, d\theta$

$$= \frac{1}{9}\int \frac{1}{u^2}\, du$$

$$= \frac{1}{9}\left[-u^{-1}\right] + C$$

$$= -\frac{1}{9}\csc\theta + C$$

$$= -\frac{1}{9}\frac{\sqrt{x^2+9}}{x} + C$$

2. Evaluate the trigonometric integral $\int \tan^3 x \sec^4 x dx$

Even power of sec:

$u = \tan x$

$du = \sec^2 x\, dx$

$\sec^2 x = 1 + \tan^2 x$

$= 1 + u^2$

$= \int \tan^3 x \cdot \sec^2 x \cdot \sec^2 x\, dx$

$= \int u^3 \cdot (1+u^2) \cdot du$

$= \frac{u^4}{4} + \frac{u^6}{6} + C$

$= \frac{\tan^4 x}{4} + \frac{\tan^6 x}{6} + C.$

3. Use partial fractions to evaluate $\int \frac{x+3}{x^2+7x+6} dx$

$\frac{x+3}{x^2+7x+6} = \frac{x+3}{(x+6)(x+1)} = \frac{A}{x+6} + \frac{B}{x+1}$

$\Rightarrow \quad x+3 = A(x+1) + B(x+6)$

$x=-1: \quad 2 = B(5) \Rightarrow B = 2/5$

$x=-6: \quad -3 = A(-5) \Rightarrow A = 3/5$

$\int \frac{x+3}{x^2+7x+6} dx = 3/5 \int \frac{1}{x+6} dx + 2/5 \int \frac{1}{x+1} dx$

$= 3/5 \ln|x+6) + 2/5 \ln|x+1|$