Name: Section: 5 6 9 10

1. Evaluate $\displaystyle\int \frac{2x+1}{(x^2+1)(x+2)}\,dx$

$$\frac{2x+1}{(x^2+1)(x+2)} = \frac{Ax+B}{x^2+1} + \frac{C}{x+2}$$ Clear fractions

$$2x+1 = (Ax+B)(x+2) + C(x^2+1)$$

$$= Ax^2 + 2Ax + Bx + 2B + Cx^2 + C.$$

$$= (A+C)x^2 + (2A+B)x + (2B+C)$$

$$\begin{cases} A + C = 0 \\ 2A + B = 2 \\ 2B + C = 1 \end{cases} \Rightarrow A = -C \Rightarrow \begin{cases} 2(-C) + B = 2 \\ 2B + C = 1 \end{cases}$$ Add 2x Eq. 2 to Eq 1.

$$\Rightarrow 5B = 4 \Rightarrow B = 4/5$$

$$\Rightarrow C = 1 - 2(4/5) = -3/5$$

$$\Rightarrow A = 3/5$$

$$= \int \frac{3/5\,x + 4/5}{x^2+1} + \frac{-3/5}{x+2}\,dx$$

$$= \frac{3}{5}\cdot\frac{1}{2}\ln(x^2+1) + \frac{4}{5}\tan^{-1}(x) - \frac{3}{5}\ln|x+2| + C.$$

2. Use the Trapezoidal Rule to find T_4 for the integral $\int_{-1}^{4} x^2 dx$

$a = -1, \quad b = 4, \quad f(x) = x^2, \quad n = 4, \quad \Delta x = \frac{b-a}{n} = \frac{4-(-1)}{4} = 1.$

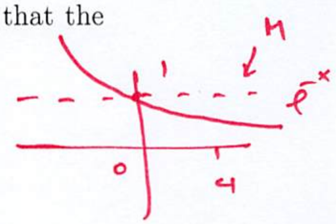
x_k	-1	0	1	2	3
$y_k = f(x_k)$	1	0	1	4	9
T_4 coeff.	1	2	2	2	1
Coeff. $\cdot\, y_k$	1	0	2	8	10

$T_4 = \frac{\Delta x}{2}(1 + 0 + 2 + 8 + 10) = \frac{1}{2}(20) = 10.$

3. The error bound for Simpson's Rule to $\int_a^b f(x)dx$ is

$$|E_S| \leq \frac{M(b-a)^5}{180n^4},$$

where M is any number such that $|f^{(4)}(x)| \leq M$ for $a \leq x \leq b$. Find n so that the approximation using Simpson's Rule S_n is within 10^{-9} of $\int_0^4 e^{-x}\, dx$.



$f^{(4)}(x) = e^{-x}$ is decreasing so is largest at $x = 0$.

We may use $M = 1$, since $|f^{(4)}(x)| = |e^{-x}| \leq 1$ for $0 \leq x \leq 4$. We want

$$\frac{M(b-a)^5}{180n^4} \leq 10^{-9}$$

$$\frac{1\cdot(4-0)^5}{180\cdot n^4} \leq 10^{-9}$$

$$\Rightarrow \frac{4^5}{180}\cdot 10^9 \leq n^4$$

$$\Rightarrow \sqrt[4]{\frac{4^5\cdot 10^9}{180}} \leq n$$

We may use the next even integer