Name: Section: 5 6 9 10

1. Evaluate the integral $\int_1^\infty \frac{3}{(2x+3)(2x+1)}\,dx$

$$\frac{3}{(2x+3)(2x+1)} = \frac{A}{2x+3} + \frac{B}{2x+1}$$

$$\Longrightarrow \quad 3 = A(2x+1) + B(2x+3)$$

$$x = -1/2 : \quad 3 = B(-1+3) \quad \Longrightarrow B = 3/2$$

$$x = -3/2 : \quad 3 = A(-3+1) \quad \Longrightarrow A = -3/2$$

$$\int \frac{3}{(2x+3)(2x+1)}\,dx = -3/2 \int \frac{dx}{2x+3} + 3/2 \int \frac{dx}{2x+1}$$

$$= -3/4 \ln|2x+3| + 3/4 \ln|2x+1| + C$$

$$\Longrightarrow \int_1^\infty \frac{3}{(2x+3)(2x+1)}\,dx = \lim_{t\to\infty} \left[\frac{3}{4}\ln|2x+1| - \frac{3}{4}\ln|2x+3|\right]_1^t$$

$$= \lim_{t\to\infty}\left[\frac{3}{4}\ln\left|\frac{2x+1}{2x+3}\right|\right]_1^t$$

$$= \frac{3}{4}\lim_{t\to\infty}\left[\ln\left|\frac{2\cancel{t}+1}{2\cancel{t}+3}\right| - \ln\left|\frac{3}{5}\right|\right]$$

$$= \frac{3}{4}\left(0 - \ln(3/5)\right) = \frac{3}{4}\ln(5/3)$$

2. Write out the first 5 values of each of the following sequences.

   (a) $\left\{\frac{2n-1}{2^n}\right\}$

   $\frac{1}{2}, \frac{3}{4}, \frac{5}{8}, \frac{7}{16}, \frac{9}{32}$

   (b) The recursive sequence given by the relation $a_{n+1} = 100 + 2a_n$ where $a_1 = 1$.

   $1, 102, 304, 708, 1516.$

3. Find the limit of the following sequences. Always use common limits whenever you can.

   (a) $\left\{\left(1-\frac{4}{n}\right)^n\right\}$

   $$\lim_{n\to\infty}\left(1-\frac{4}{n}\right)^n = \lim_{n\to\infty}\left(1+\frac{-4}{n}\right)^n = e^{-4}$$

   common limit.

   (b) $\left\{\frac{3(1-(1/2)^n)}{1-(1/2)}\right\}$

   $$\lim_{n\to\infty}\frac{3(1-(1/2)^n)}{1-(1/2)} = 6\cdot\lim_{n\to\infty}\left(1-(1/2)^n\right)$$

   $$= 6\cdot\lim_{n\to\infty} 1 - 6\cdot\lim_{n\to\infty}(1/2)^n$$

   $$= 6 - 0$$

   $$= 6.$$