Name: Section: 5 6 9 10

Determine if the following series converge or diverge. You may use any technique up to section 2.3; the techniques of geometric series, telescoping series, p-series, the divergence test, and the integral test. Find the sum if the series is a convergent geometric or telescoping series. Clearly explain your reasoning.

1. $\sum_{n=1}^{\infty} \frac{2}{n^3}$ p-series, $p = 3 > 1$

   series converges

2. $\sum_{n=1}^{\infty} \frac{2n-1}{3n-1}$ Divergence test, $\lim_{n\to\infty} \frac{2n-1}{3n-1} \overset{\text{LH}}{=} \lim_{n\to\infty} \frac{2}{3} = \frac{2}{3} \neq 0$

   series diverges

3. $\sum_{n=2}^{\infty} \frac{2^n}{5^{n-2}}$ $= \sum_{n=2}^{\infty} 25 \cdot \left(\frac{2}{5}\right)^n$ geometric, $|r| = \left|\frac{2}{5}\right| < 1$

   series converges sum $= \frac{4}{1 - 2/5} = \frac{20}{3}$

4. $\sum_{n=1}^{\infty} \frac{1}{\sqrt[5]{n}}$ p-series, $p = 1/5 \leq 1$

   series diverges

5. $\sum_{n=1}^{\infty} (1.1)^n$ geometric, $|r| = |1.1| \geq 1$

   series diverges

6. $\sum_{n=1}^{\infty} \frac{1}{1+n^2}$ Integral test,

$\int_1^\infty \frac{1}{1+x^2}dx = \lim_{t\to\infty}\int_1^t \frac{1}{1+x^2}dx = \lim_{t\to\infty} \arctan x\Big|_1^t$

$= \lim_{t\to\infty}(\arctan t - \pi/4)$

$= \pi/4$

Series converges

7. $\sum_{n=1}^{\infty} \left(1+\frac{2}{n}\right)^n$ Divergence test

$\lim_{n\to\infty}\left(1+\frac{2}{n}\right)^n = e^2 \neq 0$

Series diverges

8. $\sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}}\right)$ telescoping,

$= \lim_{n\to\infty}\left(1 - \frac{1}{\sqrt{n+1}}\right) = 1$

Series converges

9. $\sum_{n=2}^{\infty} \frac{1}{n\ln n}$ Integral test,

$u = \ln x \quad du = \frac{1}{x}dx$

$\int_2^\infty \frac{1}{x\ln x}dx = \lim_{t\to\infty}\int_2^t \frac{1}{x \cdot \ln x}dx = \lim_{t\to\infty}\int_{\ln 2}^{\ln t}\frac{1}{u}du$

$= \lim_{t\to\infty}\left[\ln(\ln t) - \ln(\ln 2)\right]$

$= \infty$

Series diverges