Name: Section: 5 6 9 10

Determine if the following series converge or diverege. You may use techniques of geometric series, telescoping series, p-series, divergence test, integral test, comparison test, limit comparison test, alternating series test, absolute convergence test. Show your work and clearly indicate which test you are using.

1. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}+1}$. Alternating series $(-1)^n b_n$ where $b_n = \frac{1}{\sqrt{n}+1}$

yes $b_n = \frac{1}{\sqrt{n}+1}$ are positive and decreasing

and

$$\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{1}{\sqrt{n}+1} = 0$$

$\therefore$ Series converges by A.S.T.

2. $\sum_{n=1}^{\infty} \frac{1}{3^n+n}$. Comparison test:

$$\frac{1}{3^n+n} \le \frac{1}{3^n}$$

Since $\sum_{n=1}^{\infty} \frac{1}{3^n}$ converges (geometric $|r| = |\frac{1}{3}| < 1$)

then $\sum_{n=1}^{\infty} \frac{1}{3^n+n}$ converges

3. $\sum_{n=2}^{\infty} (-1)^n \frac{n}{n-1}$. alternating series $(-1)^n b_n$ with $b_n = \frac{n}{n-1}$

$b_n = \frac{n}{n-1}$ positive and decreasing, but

$$\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{n}{n-1} = 1$$

therefore, $\lim_{n\to\infty} (-1)^n \frac{n}{n-1}$ DNE $(\neq 0)$

$\Longrightarrow$ series diverges by Divergence test

4. $\sum_{n=2}^{\infty} \frac{\sin(n) + 2\cos(n)}{n^2 + \ln(n)}$ ← sign of numerator unpredictable try A.C.T.

$$\left| \frac{\sin(n) + 2\cos(n)}{n^2 + \ln(n)} \right| \le \frac{3}{n^2}$$

Since $\sum_{n=2}^{\infty} \frac{3}{n^2}$ converges (p-series $p = 2 > 1$)

$\Rightarrow \sum_{n=2}^{\infty} \left| \frac{\sin n + 2\cos n}{n^2 + \ln(n)} \right|$ converges by comparison

$\Rightarrow \sum_{n=2}^{\infty} \frac{\sin n + 2\cos n}{n^2 + \ln(n)}$ converges absolutely

5. $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n} - 1}$

Comparison test:

$$\frac{1}{\sqrt{n} - 1} \ge \frac{1}{\sqrt{n}}$$

Since $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}}$ diverges (p-series $p = 1/2 \le 1$)

$\Rightarrow$ series converges

6. $\sum_{n=1}^{\infty} \frac{2n - 1}{n^3 - 9}$. limit compare with $\frac{2n}{n^3} = \frac{2}{n^2}$

$$\lim_{n \to \infty} \frac{2n - 1}{n^3 - 9} \cdot \frac{n^2}{2} = \lim_{n \to \infty} \frac{2n^3 - n^2}{2n^3 - 18}$$

$$= \lim_{n \to \infty} \frac{2 - 1/n}{2 - 18/n}$$

$$= 1 > 0$$

Since $\sum_{n=1}^{\infty} \frac{2}{n^2}$ converges (p-series $p = 2 > 1$)

then $\sum_{n=1}^{\infty} \frac{2n-1}{n^3 - 9}$ converges