Name: Solutions Section: 5 6 9 10

Determine if the following series converge or diverge. You may use any technique we've covered. Show your work and clearly state which test you are using.

1. $\displaystyle\sum_{n=1}^{\infty}\left(\frac{n-2}{2n}\right)^{2n}$ ← n^{th} power, try root test

$$\sqrt[n]{|a_n|} = \sqrt[n]{\left|\left(\frac{n-2}{2n}\right)^{2n}\right|} = \left(\frac{n-2}{2n}\right)^2 \longrightarrow \frac{1}{4} < 1$$

Series converges

2. $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2 7^n}$ ← alternating, try AST

$$b_n = \frac{1}{n^2 \cdot 7^n}$$

yes, the b_n's are positive & are decreasing, and

$$\lim_{n\to\infty}\frac{1}{n^2\cdot 7^n} = 0$$

Series converges

3. $\displaystyle\sum_{n=1}^{\infty}\frac{(-3)^{n+1}}{2^{2n}} = \sum_{n=1}^{\infty}(-3)\left(-\frac{3}{4}\right)^n$ geometric

$$|\text{ratio}| = \left|-\frac{3}{4}\right| < 1$$

Series converges.

4. $\displaystyle\sum_{n=1}^{\infty} \frac{7^n}{(2n)!}$ factorial, try ratio test

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{7^{n+1}}{(2(n+1))!}\cdot\frac{(2n)!}{7^n}\right| = \frac{7^n\cdot 7}{7^n}\cdot\frac{(2n)!}{(2n+2)(2n+1)(2n)!}$$

$$= \frac{7}{(2n+2)(2n+1)} \to 0 < 1$$

Series converges

5. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^5+1}}$

$$\frac{1}{\sqrt[3]{n^5+1}} \leq \frac{1}{\sqrt[3]{n^5}} = \frac{1}{n^{5/3}}$$

Since $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^{5/3}}$ converges (p-series $p = 5/3$), then

$\displaystyle\sum_{n=1}^{\infty}\frac{1}{\sqrt[3]{n^5+1}}$ converges

6. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n-3}{n}\right)^n$ n^{th} power, try root test

$$\sqrt[n]{|a_n|} = \sqrt[n]{\left(\frac{n-3}{n}\right)^n} = \frac{n-3}{n} \to 1 \quad \text{inconclusive}$$

note

$$\lim_{n\to\infty}\left(\frac{n-3}{n}\right)^n = e^{-3} \neq 0$$

Series diverges by divergence test.