3. $\displaystyle\sum_{n=1}^{\infty} \frac{(x-1)^n}{2^n\sqrt{n}}$ Let $a_n = \displaystyle\sum_{n=1}^{\infty} \frac{(x-1)^n}{2^n\sqrt{n}}$. Try root test

$$\sqrt[n]{|a_n|} = \sqrt[n]{\frac{|x-1|^n}{2^n \cdot n^{1/2}}} = \frac{|x-1|}{2\cdot\left(\sqrt[n]{n}\right)^{1/2}} \longrightarrow \frac{|x-1|}{2}$$

$$\frac{|x-1|}{2} < 1 \iff |x-1| < 2 \iff -2 < x-1 < 2$$

$$\iff -1 < x < 3$$

so $R = 2$.

$x = 3$: $\displaystyle\sum a_n = \sum_{n=1}^{\infty} \frac{(3-1)^n}{2^n\cdot\sqrt{n}} = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ diverges

(p-series $p = 1/2$)

$x = -1$: $\displaystyle\sum a_n = \sum_{n=1}^{\infty} \frac{(-1-1)^n}{2^n\cdot\sqrt{n}} = \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ this series

doesn't converge absolutely (see above). Test $\sum a_n$

with AST: $b_n = \frac{1}{\sqrt{n}}$

(i) yes, b_n's are positive and decreasing

(ii) $\displaystyle\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$

$\therefore$ series converges conditionally.

Abs. conv.: $-1 < x < 3$ IOC: $-1 \le x < 3$

cond. conv: $x = -1$ ROC: 2

diverges: elsewhere

Name: Section: 5 6 9 10

For the following power series, determine for which x does the series converge absolutely, converge conditionally, and diverge. State the radius of convergence and the interval of convergence.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n x^n}{n^2}$ Let $a_n = \dfrac{2^n x^n}{n^2}$. Apply root test

$$\sqrt[n]{|a_n|} = \sqrt[n]{\frac{2^n |x|^n}{n^2}} = \frac{2|x|}{(\sqrt[n]{n})^2} \longrightarrow \frac{2|x|}{(1)^2} = 2|x|$$

$2|x| < 1 \iff |x| < 1/2 \iff -1/2 < x < 1/2$ so $R = 1/2$

$x = 1/2$: $\displaystyle\sum a_n = \sum_{n=1}^{\infty} \frac{2^n (1/2)^n}{n^2} = \sum_{n=1}^{\infty} \frac{1}{n^2}$ converges absolutely

(p-series $p = 2$)

$x = -1/2$: $\displaystyle\sum a_n = \sum_{n=1}^{\infty} \frac{2^n (-1/2)^n}{n^2} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ converges absolutely (see above)

Abs conv. : $-1/2 \le x \le 1/2$

cond conv: nowhere

diverges : $x < -1/2$ or $x > 1/2$

I.O.C. : $-1/2 \le x \le 1/2$

R.O.C : $1/2$

2. $\displaystyle\sum_{n=0}^{\infty} \frac{2^n x^n}{n!}$ Let $a_n = \dfrac{2^n x^n}{n!}$. Try Ratio test

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{2^{n+1} \cdot x^{n+1}}{(n+1)!} \cdot \frac{n!}{2^n \cdot x^n}\right| = \frac{2 \cdot |x|}{n+1} \longrightarrow 0$$

Absolute conv. : $-\infty < x < \infty$

cond. conv : nowhere

diverges : nowhere

I.O.C : $-\infty < x < \infty$

R.O.C : $R = \infty$