Name: Section: 5 6 9 10

1. Find the Taylor polynomial of order 3 for each of the following functions at the specified center.

   (a) $f(x) = \sqrt{1+x}$ at $a = 0$.

$f(x) = (1+x)^{1/2} \Rightarrow f(0) = 1$

$f'(x) = \frac{1}{2}(1+x)^{-1/2} \Rightarrow f'(0) = 1/2$

$f''(x) = -1/4\,(1+x)^{-3/2} \Rightarrow f''(0) = -1/4$

$f'''(x) = \frac{3}{8}(1+x)^{-5/2} \Rightarrow f'''(0) = 3/8$

$T_3(x) = 1 + \frac{1}{2}x + \frac{-1/4}{2!}x^2 + \frac{3/8}{3!}x^3$

   (b) $f(x) = \sin(2x)$ at $a = \pi/3$.

$f(x) = \sin(2x) \Rightarrow f(\pi/3) = \sin(2\pi/3) = \frac{\sqrt{3}}{2}$

$f'(x) = 2\cos(2x) \Rightarrow f'(\pi/3) = 2\cos(2\pi/3) = 1$

$f''(x) = -4\sin(2x) \Rightarrow f''(\pi/3) = -4\sin(\frac{2\pi}{3}) = -2\sqrt{3}$

$f'''(x) = -8\cos(2x) \rightarrow f'''(\pi/3) = -8\cos(2\pi/3) = -4$

$T_3(x) = \frac{\sqrt{3}}{2} + 1(x-\pi/3) + \frac{-2\sqrt{3}}{2!}(x-\pi/2)^2 + \frac{-4}{3!}(x-\pi/3)^3$

2. Using the Maclauring series

$$\cos(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!},$$

(a) Find a Maclaurin series for $x^3 \cos(2x)$

(b) Find the sum of the series

$$1 - \frac{\pi^2}{6^2 \cdot 2!} + \frac{\pi^4}{6^4 \cdot 4!} - \frac{\pi^6}{6^6 \cdot 6!} + \cdots$$

(a) $\cos(2x) = \sum_{n=0}^{\infty} (-1)^n \frac{(2x)^{2n}}{(2n)!} = \sum_{n=0}^{\infty} (-1)^n \frac{2^{2n} x^{2n}}{(2n)!}$

$x^3 \cos(2x) = x^3 \sum_{n=0}^{\infty} (-1)^n \frac{2^{2n} x^{2n}}{(2n)!}$

$= \sum_{n=0}^{\infty} (-1)^n \frac{2^{2n} x^3 x^{2n}}{(2n)!}$

$= \sum_{n=0}^{\infty} (-1)^n \frac{2^{2n} x^{2n+3}}{(2n)!}$

(b) sum $= \sum_{n=0}^{\infty} (-1)^n \frac{(\pi/6)^{2n}}{(2n)!} = \cos(\pi/6) = \frac{\sqrt{3}}{2}$