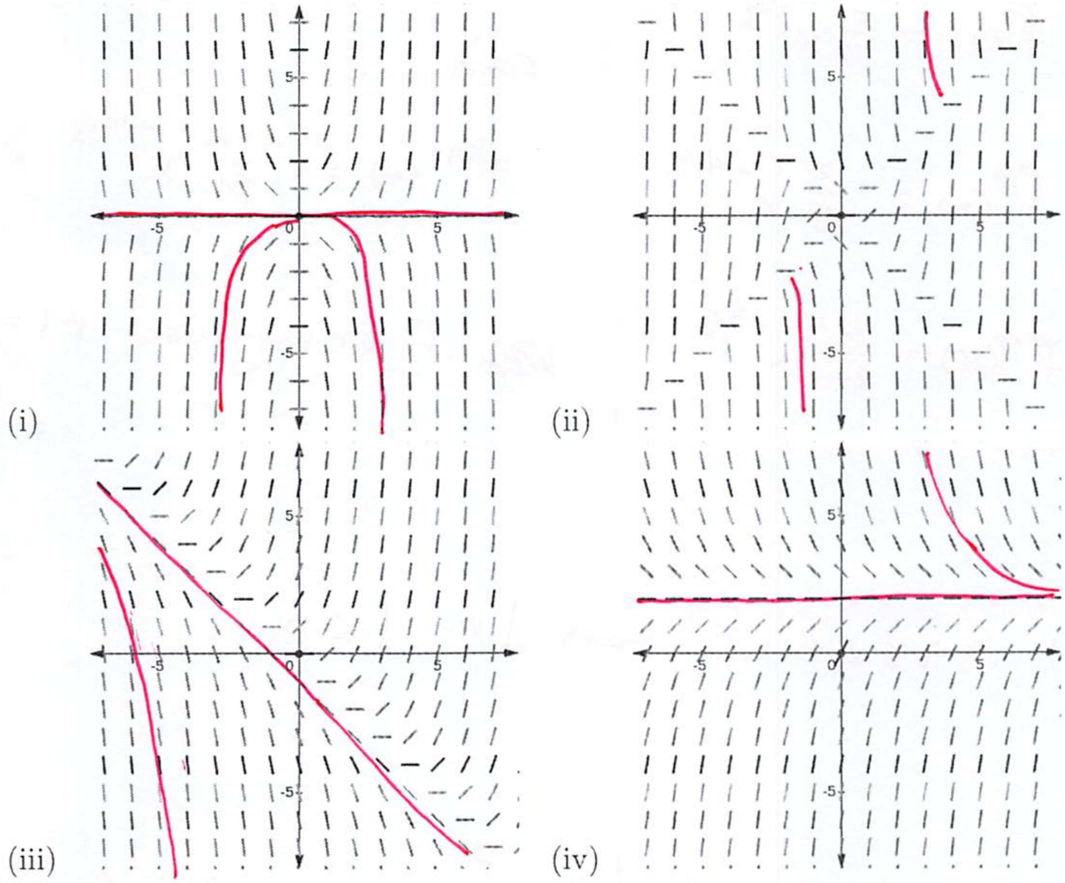
Name: Section: 5 6 9 10

1. Match the differential equation with its direction field. Draw two solution curves on each direction field.

   $y' = 2 - y$ (iv)
   $y' = xy$ (i)
   $y' = x + y$ (iii)
   $y' = x^2 - y^2$ (ii)



2. Show that $y = e^{-2x}$ is a solution to the differential equation

$$y'' + y' - 2y = 0$$

$$\frac{d^2}{dx^2}\left[e^{-2x}\right] + \frac{d}{dx}\left[e^{-2x}\right] - 2e^{-2x}$$

$$= \frac{d}{dx}\left[-2e^{-2x}\right] - 2e^{-2x} - 2e^{-2x}$$

$$= 4e^{-2x} - 4e^{-2x} = 0.$$

3. Use Taylor's inequality to find an upper bound on the maximum error in using $T_2(x)$ centered at $a = 1$ to approximate $f(x) = x^{2/3}$ over $0.9 \le x \le 1.1$.

$f(x) = x^{2/3}$

$f'(x) = \frac{2}{3} x^{-1/3}$

$f''(x) = -\frac{2}{9} x^{-4/3}$

$f'''(x) = \frac{8}{27} x^{-7/3}$

(1) f''' is positive on $0.9 \le x \le 1.1$

and f''' is decreasing on $0.9 \le x \le 1.1$

since

$f^{(4)}(x) = \frac{-56}{81} x^{-10/3} < 0$

So we may use $M = f'''(0.9)$

$= \frac{8}{27}(0.9)^{7/3}$

(2) $0.9 \le x \le 1.1 \iff |x-1| \le 0.1$

So

$$|R_2(x)| \le \frac{M|x-1|^3}{3!} < \frac{8}{27}(0.9)^{7/3} \cdot \frac{(0.1)^3}{3!}$$