(2) $f(x) = x^2 - 2x + 1$ for $x \geq 1$

Solve $f(y) = x$ for y

$$y^2 - 2y + 1 = x$$

$$y^2 - 2y + (1-x) = 0$$

$a = 1$

$b = -2$

$c = 1 - x$

$$y = \frac{2 \pm \sqrt{4 - 4(1)(1-x)}}{2(1)}$$

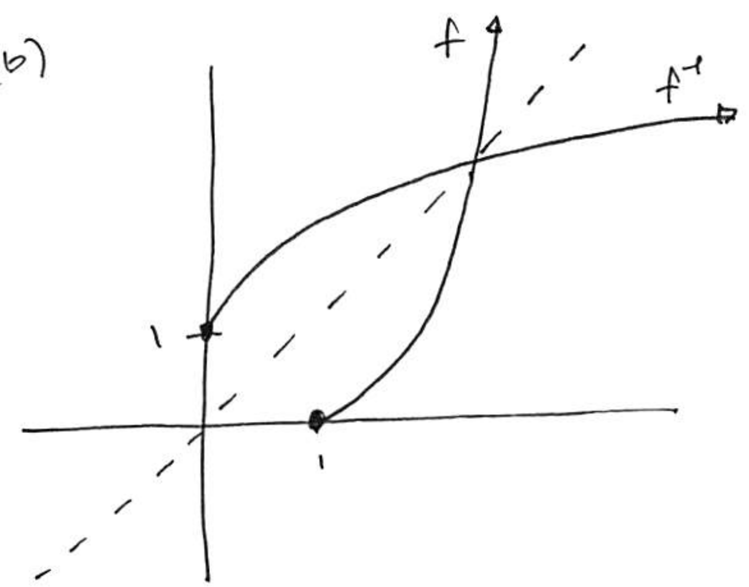
$$= 1 \pm \sqrt{1 - 1 + x}$$

$$= 1 \pm \sqrt{x}$$

Then $y = 1 + \sqrt{x}$ as $y \geq 1$

(a) $f^{-1}(x) = 1 + \sqrt{x}$

(b)



(c)

dom f^{-1} = $[0, \infty)$

ran f^{-1} = $[1, \infty)$

(4) $f(x) = 4x^2$ for $x \geq 0$

(a) Solve $f(y) = x$ for y

$$4y^2 = x$$

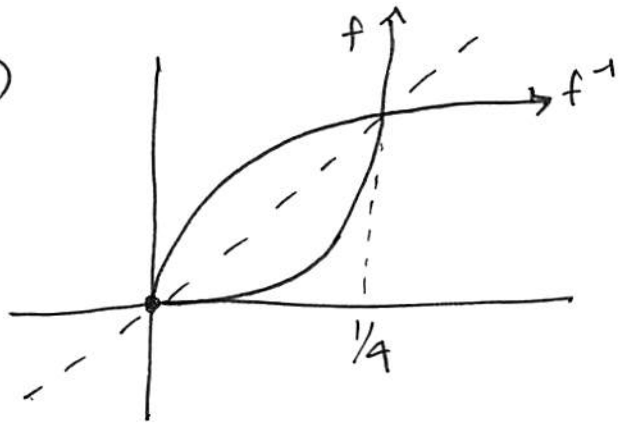
$$y^2 = \frac{x}{4}$$

$$y = \pm \frac{\sqrt{x}}{2}$$

But since $y \geq 0$, then $y = \frac{\sqrt{x}}{2}$

$$\therefore \quad f^{-1}(x) = \frac{\sqrt{x}}{2}$$

(b)



(c) $(f^{-1})'(x) = \dfrac{1}{4\sqrt{x}} \implies (f^{-1})'(f(5)) = (f^{-1})'(100)$

$$= \frac{1}{4\sqrt{100}}$$

$$= \frac{1}{40}$$

$f'(x) = 8x \implies f'(5) = 8 \cdot 5 = 40$

$$\therefore \quad \left.\frac{df^{-1}}{dx}\right|_{x=f(5)} = \frac{1}{40} = \frac{1}{\left.\frac{df}{dx}\right|_{x=5}}$$

(5) Skip.

(6) Since g is invertible and passes through the origin (which lies on the diagonal $y = x$) so does g^{-1}.

$$g(0) = 0 \iff 0 = g^{-1}(0)$$

We are given that $g'(0) = 3$. Then the FFT says

$$(g^{-1})'(0) = \frac{1}{g'(g^{-1}(0))} = \frac{1}{g'(0)} = \frac{1}{3}$$

(7) Let $f(x) = (x-1)^3$

(a) $f'(x) = 3(x-1)^2$ is positive for all real numbers $x \neq 1$. Thus, f is increasing

(b) $$(y-1)^3 = x$$
$$y - 1 = x^{1/3}$$
$$y = x^{1/3} + 1$$
$$\therefore f^{-1}(x) = x^{1/3} + 1.$$

(c) $f'(x) = 3(x-1)^2$

$3(x-1)^2 = 0 \iff x = 1$

(d) Excluding the pt $x=1$ from the domain allows us to apply the IFT

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$

$$= \frac{1}{3(f^{-1}(x) - 1)^2}$$

$$= \frac{1}{3((x^{1/3}+1) - 1)^2}$$

$$= \frac{1}{3x^{2/3}}$$

(e) $$(f^{-1})'(x) = \frac{d}{dx}(x^{1/3}+1)$$

$$= \tfrac{1}{3}x^{-2/3}$$

$$= \frac{1}{3x^{2/3}}$$

(8)

(b) $y = \ln x^3 = 3 \ln x$

$\longrightarrow y' = \frac{3}{x}$

(f) $y = \log_5 e^x = \frac{\ln e^x}{\ln 5} = \frac{x}{\ln 5}$

$\longrightarrow y' = \frac{1}{\ln 5}$

(g) $y = \ln(3\theta e^{-\theta}) = \ln 3 + \ln\theta + \ln e^{-\theta}$

$= \ln 3 + \ln\theta - \theta$

$\longrightarrow y' = \frac{1}{\theta} - 1$

(a)

(e) $$\int \frac{dx}{2\sqrt{x} + 2x} = \int \frac{dx}{2\sqrt{x}(1+\sqrt{x})}$$

$u = 1+\sqrt{x}$

$du = \frac{1}{2\sqrt{x}}\,dx$

$$= \int \frac{du}{u}$$

$$= \ln|u| + C = \ln|1+\sqrt{x}| + C$$

(f) $$\int \frac{dx}{1+e^x} = \int \frac{1+e^x-e^x}{1+e^x}\,dx$$

$$= \int 1 - \frac{e^x}{1+e^x}\,dx$$

$u = 1+e^x$

$du = e^x dx$

$$= \int dx - \int \frac{du}{u}$$

$$= x - \ln|u| + C$$

$$= x - \ln|1+e^x| + C.$$

(f) $\displaystyle\int_1^{e^x} \frac{1}{t}\,dt = \ln e^x = x$

(10) (a) $y = \tan\theta\sqrt{2\theta+1}$

$\Longrightarrow \ln y = \ln\left(\tan\theta\sqrt{2\theta+1}\right)$

$= \ln\tan\theta + \frac{1}{2}\ln(2\theta+1)$

$\Longrightarrow \dfrac{y'}{y} = \dfrac{1}{\tan\theta}\dfrac{d}{dx}\tan\theta + \dfrac{1}{2}\dfrac{1}{2\theta+1}\dfrac{d}{dx}(2\theta+1)$

$= \dfrac{\sec^2\theta}{\tan\theta} + \dfrac{1}{2\theta+1}$

$\Longrightarrow y' = y\left(\dfrac{\sec^2\theta}{\tan\theta} + \dfrac{1}{2\theta+1}\right)$

$= \tan\theta\sqrt{2\theta+1}\left(\dfrac{\sec^2\theta}{\tan\theta} + \dfrac{1}{2\theta+1}\right)$

$= \sec^2\theta\sqrt{2\theta+1} + \dfrac{\tan\theta}{\sqrt{2\theta+1}}$

(11) $y' = e^t \sin(e^t - 2)$; $y(\ln 2) = 0$.

$$y = \int e^t \sin(e^t - 2)\, dt \qquad u = e^t - 2$$
$$du = e^t dt$$

$$= \int \sin u\, du$$

$$= -\cos(e^t - 2) + C$$

$$y(\ln 2) = 0 \iff -\cos(e^{\ln 2} - 2) + C = 0$$
$$\iff C = \cos(2-2)$$
$$\iff C = \cos(0) = 1$$

$$\therefore y = -\cos(e^t - 2) + 1$$