Math 242 Homework 5: due 7/15

1. 9.2 1-58; 9.3 1-28; 9.4 1-36. Do every other even (2, 6, 10,...). Skip any problem that has a hyperbolic function like sinh, tanh, etc.

2. Below are the n-th partial sums of a series $\sum a_n$. If it exists, what is $\sum a_n$?

   (a) $s_n = \dfrac{e^{2n}}{4^n}$ (b) $s_n = \dfrac{\sqrt[n]{4^{n^2} n}}{n!}$

3. Five geniuses divide a pizza into sixths and each of them gets a piece. They then divide the remaining sixth again into sixths, and each gets another piece. Show that if they continue this process indefinitely, then each of them will get a fifth of the original pizza.

4. Show that the sum of the deleted segments in the construction of the Cantor set C is 1 (see homework 4). That is, if a_n is the length of the segment deleted from C_{n-1}, then $\sum a_n = 1$. This makes sense because in the last homework we showed that C has length 0, and it should be the case that

   $$\text{length of stuff not in } C = \text{length of } [0,1] \text{ - length of } C.$$

5. Let $\{a_n\}$ be a sequence of integers such that $0 \leq a_n \leq 9$. Show that $\sum a_n 10^{-n}$ converges. This is the decimal number we usually denote by $0.a_1a_2a_3a_4\ldots$

6. Show that $\sum_{n=1}^{\infty} \frac{1}{n!}$ converges using the bounded sum test.

7. Show that $\sum_{n=0}^{\infty} e^{-n^2}$ converges using the integral test.