Math 242 Homework 6: due 7/22

1. 9.5 1-46; 9.6 1-48; 9.7 1-38. Do every other even. Skip any problem with a hyperbolic function.

2. In class we showed that the power series

$$y = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

converges for all real numbers x.

   (a) Show that $y(0) = 1$ and that $y' = y$.

   (b) Solve the differential equation $y' = y$ with initial condition $y(0) = 1$. Since the solution to this differential equation is unique, what can we conclude?

3. Show that

$$y = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

is a solution to the differential equation $y'' + y = 0$.

4. Fun with pi and series.

   (a) A movie came out earlier this year about the highly regarded mathematician Srinivasa Ramanujan. One of many formulas he discovered was

$$\frac{1}{\pi} = \frac{\sqrt{8}}{9801} \sum_{n=0}^{\infty} \frac{(4n)!(1103 + 26390n)}{(n!)^4 396^{4n}}.$$

   Show that the above series indeed converges.

   (b) Write π as an infinite series using the power series representation for $\tan^{-1} x$.

   (c) Repeat the above exercise using the power series representation for e^x. [*Hint*: the series represention of e^x is the one in problem 2.]

5. In this problem we will show that $\sum n = 1 + 2 + 3 + 4 + 5 + \cdots = -1/12$. Makes sense right?

   (a) Find a power series representation for $1/(1+x)^2$. Evaluate this power series at $x = 1$ and write out a few terms to see what it looks like. [*Hint*: see example 4 in section 9.7]

   (b) Let $S = 1+2+3+4+5+\cdots$. Compute $4S$, line up its terms beneath the even terms of S, and substract the two series. This should give you an expression for $-3S$.

   (c) Conclude from the previous two parts that $S = -1/12$.

Note here that this does not mean that the series $\sum n$ converges to $-1/12$. According to the our definition of convergence this series does not converge (why?). What we did here was manipulate the series in a way where we could extract a value.