MATH 242 Summer 2016
Practice Final

Name: ____________________

Instructions:

- Begin by writing your name in the space above.
- You have 80 minutes to complete this exam.
- No phones, calculators, notes, or any form of assistance may be used during the exam.
- You must show all of your work, unless you are asked not to. Answers which are cryptic or have no supporting evidence will most likely not receive full credit. When in doubt, ask.
- Please be organized! Answer questions in the space provided as neatly as possible. If you run out of room, continue on a piece of scratch paper and make a clear note of it.

Question	Points	Score
1	10	
2	10	
3	20	
4	15	
5	15	
6	30	
Total:	100	

1. (10 points) True/False. Circle your answer. You do not need to show work.

(a) True False Suppose f is differentiable and invertible. If the derivative of f has slope 2 at the point $(1, 0)$, then f^{-1} has slope $1/2$ at $(1, 0)$.

(b) True False Every differentiable function is invertible.

(c) True False A sequence a is a function $a : \mathbb{N} \to \mathbb{R}$.

(d) True False If $\{a_n\}$ is a decreasing sequence such that $|a_n| < M$ for some positive M, then $\{a_n\}$ has a limit.

(e) True False For any $a \in \mathbb{R}$, $\lim\limits_{x \to \infty} \frac{x^a}{e^x} = 0$.

(f) True False Suppose f, g are nonnegative functions defined at every real number with $f(x) \leq g(x)$ for all x. Then $\sum f(n)$ converges whenever $\sum g(n)$ converges.

(g) True False A Taylor polynomial of order n generated by f at $x = 1$ is the n-th partial sum of the Taylor series generated by f at $x = 1$.

(h) True False If a sequence is unbounded, then the even terms in the sequence must be unbounded.

(i) True False The n-th term test can be used to prove the convergence of a series.

(j) True False If f is an odd function and $\displaystyle\int_0^{\infty} f(x)dx$ diverges, then $\displaystyle\int_{-\infty}^{\infty} f(x)dx$ diverges.

2. Improper integrals and indeterminate forms. *Warning*: writing nonsense such as e^{∞}, $\ln x|_0^{\infty}$, $\infty - \infty = 0$, etc. will result in 0 points for that problem.

   (a) (5 points) Evaluate $\displaystyle\int_{-1}^{0} \frac{dx}{2x+1}$.

   (b) (5 points) Compute the limit $\displaystyle\lim_{x \to 0^+} (x^x)^x$.

3. Integration.

   (a) (5 points) Use Simpson's with $n = 4$ to approximate $\int_0^2 (t^3 + t)dt$. What is an upper bound for the error?

(b) (5 points) Find $\int \frac{\sqrt{9-\theta^2}}{\theta} d\theta$.

(c) (5 points) Find $\int (\ln y)^2 dy$

(d) (5 points) Find $\int \frac{dz}{z^4 - 16}$

4. Series.

(a) (5 points) Find the sum $\sum_{n=2}^{\infty}(-1)^n 2^{n+1}\pi^{-n}$.

(b) (5 points) Determine whether $\sum_{n=2}^{\infty} \frac{1}{(\ln n)^2}$ diverges or converges.

(c) (5 points) Classify $\sum_{n=1}^{\infty}(-1)^n \frac{4^n + n}{n!}$ as absolutely convergent, conditionally convergent, or divergent.

5. Power series.

(a) (5 points) Find the Maclaurin series for $\dfrac{1}{1-x^4}$.

(b) (5 points) Represent $\int \cos x^2 dx$ as a power series.

(c) (5 points) Find the interval of convergence for the power series $\sum_{n=0}^{\infty} \frac{2^n x^{2n}}{(2n)!}$.

6. Differential equations.

(a) (5 points) Show that $y = x(\ln x + C)$ is a solution to the differential equation $x + y - xy' = 0$.

(b) (5 points) Solve $y'' + 16y = 0$.

(c) (8 points) Find the genral solution to the equation $y'' - y = e^x$.

(d) A tank contains 100 L of water. A solution with a salt concentration of 0.4 kg/L is added at a rate of 5 L/min. The solution is well-mixed and is drained from the tank at a rate of 3 L/min. Let $y = y(t)$ be the amount of salt in the tank at time t.

i. (5 points) Show that the differential equation which models the amount of salt in the tank is

$$y' = 2 - \frac{3y}{100 - 2t}.$$

ii. (5 points) Show that the equation which models the amount of salt in the tank is

$$y(t) = \frac{2}{5}(100 + 2t)^{1/2} - 4000(100 + 2t)^{-3/2}.$$

iii. (2 points) What is amount of salt in the tank after 20 mins?