MATH 242 Summer 2016
Practice Exam 2

Name: ____

Instructions:

- Begin by writing your name in the space above.
- You have 80 minutes to complete this exam.
- No phones, calculators, notes, or any form of assistance may be used during the exam.
- You must show all of your work, unless you are asked not to. Answers which are cryptic or have no supporting evidence will most likely not receive full credit. When in doubt, ask.
- Please be organized! Answer questions in the space provided as neatly as possible. If you run out of room, continue on a piece of scratch paper and make a clear note of it.

Question	Points	Score
1	7	
2	10	
3	17	
4	20	
5	30	
6	22	
Total:	106	

1. (7 points) True/False. Circle your answer. You do not need to show work. Each correct answer is worth 1 point, an incorrect answer is worth -1.5 points. If you do not want to be marked on any problem write the symbol "Z" next to the problem; if you do this, then you will neither gain or lose points.

(a) True False If $\sum a_n$ is a convergent series with nonnegative terms, then $\sum(-1)^n a_n$ may diverge.

(b) True False If $\{a_n\}$ and $\{b_n\}$ both diverge, then $\{a_n + b_n\}$ must diverge.

(c) True False If the even terms in a sequence converge to 0 and the odd terms converge to 1, then the entire sequence converges to the average 1/2.

(d) True False Since $\lim_{n\to\infty} \frac{1}{\ln n} = 0$, then the series $\sum_{n=1}^{\infty} \frac{1}{\ln n}$ converges.

(e) True False If $0 \leq a_n \leq b_n$ for all $n > N$ (N some integer) and the series $\sum b_n$ diverges, then the series $\sum a_n$ diverges.

(f) True False If $\sum a_n^2$ diverges, then $\sum a_n$ diverges.

(g) True False If $a_n \leq b_n \leq c_n$ and the limits $\lim_{n\to\infty} a_n$ and $\lim_{n\to\infty} c_n$ both exist, then the limit $\lim_{n\to\infty} b_n$ exists.

2. Sequences.

   (a) (2 points) State the monotone sequence theorem.

   (b) (3 points) Give an example of sequence which is *not* monotone and converges to $-1/2$.

   (c) (5 points) Find the limit of the sequence $\{|\sin n|(\ln n)^{-1}\}_{n=1}^{\infty}$

3. Series.

   (a) (2 points) State the bounded sum test.

   (b) (5 points) Find $\sum a_n$ if its n-th partial sum is $1 + \dfrac{1}{n(n+1)}$.

(c) (5 points) Find $\sum_{n=1}^{\infty} 4^{1-n}\pi^n$.

(d) (5 points) Find $\sum_{n=2}^{\infty} \frac{-2}{n^2-1}$.

4. Series Continued. Determine whether the following series converge or diverge.

(a) (5 points) $\displaystyle\sum_{n=1}^{\infty} \frac{(n+3)!}{3!n!3^n}$

(b) (5 points) $\displaystyle\sum_{n=1}^{\infty} n\sin(1/n)$

(c) (5 points) $\sum_{n=1}^{\infty}(-1)^n(n^{-2}+2^{-n})$

(d) (5 points) $\sum_{n=1}^{\infty}\frac{3n^2+1}{n^3-4}$

5. Power series.

(a) (10 points) Find a power series representation for $\dfrac{e^x + e^{-x}}{2}$.

(b) (10 points) Use power series to compute the limit $\lim_{x\to\infty} \dfrac{(\sin x)/x - \cos x}{x^2}$.

(c) (10 points) Find the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(3x-1)^n}{n}$.

6. Taylor series.

(a) (2 points) State Taylor's theroem.

(b) (10 points) Find the Taylor series generated by $f(x) = \ln x$ at $x = 2$.

(c) (10 points) Find the Maclaurin series generated by $f(x) = e^{2x}$.