

MATH 242 Summer 2016

Practice Exam 2

Name: _____

Instructions:

- Begin by writing your name in the space above.
- You have 80 minutes to complete this exam.
- No phones, calculators, notes, or any form of assistance may be used during the exam.
- You must show all of your work, unless you are asked not to. Answers which are cryptic or have no supporting evidence will most likely not receive full credit. When in doubt, ask.
- Please be organized! Answer questions in the space provided as neatly as possible. If you run out of room, continue on a piece of scratch paper and make a clear note of it.

Question	Points	Score
1	7	
2	10	
3	17	
4	20	
5	30	
6	22	
Total:	106	

1. (7 points) True/False. Circle your answer. You do not need to show work. Each correct answer is worth 1 point, an incorrect answer is worth -1.5 points. If you do not want to be marked on any problem write the symbol "Z" next to the problem; if you do this, then you will neither gain or lose points.

- (a) True False Maclaurin series always converge to the generating function.
(b) True False $e^{-i\pi} = -1$.
(c) True False If the sequence of n -th partial sums of a series with nonnegative terms is bounded, then the series converges.
(d) True False If $\sum a_n$ diverges, then its sequence of n -th partial sums is unbounded.
(e) True False If $\sum a_n$ converges, then $a_n \rightarrow 0$.
(f) True False If $\{a_n\}$ diverges, then $\{a_n/n\}$ diverges.
(g) True False If $\{a_n\}$ is a sequence converging to a real number L , then

$$\lim_{n \rightarrow \infty} \cos(\ln(|a_n| + 1)) = \cos(\ln(|L| + 1)).$$

- (h) True False A power series may converge at exactly two distinct real numbers.

2. Sequences.

(a) (2 points) What does it mean when we say “ f interpolates the sequence $\{a_n\}$ ”?

(b) (3 points) Give an example of a monotone sequence that converges to -1 .

(c) (5 points) Find the limit of the sequence $a_n = (e^{e^n})^{1/n!}$.

3. Series.

(a) (2 points) What is the formal definition of infinite series.

(b) (5 points) Show that $0.\bar{2} = 2/9$ using geometric series.

(c) (5 points) Find $\sum a_n$ if its n -th partial sum is $\frac{2^n - 1}{2^n - 2^{n-1}}$.

(d) (5 points) Find $\sum_{n=1}^{\infty} \left(\frac{1}{2n-1} - \frac{1}{2n+1} \right)$.

4. Series.

(a) (5 points) Use the integral test to determine whether $\sum_{n=1}^{\infty} \frac{1}{4n^2 + 1}$ converges or diverges.

(b) (5 points) Classify $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n + \sqrt{n}}$ as absolutely convergent, conditionally convergent, or divergent.

(c) (5 points) Does the series $\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^n$ converge or diverge?

(d) (5 points) Does the series $\sum_{n=1}^{\infty} \left(\frac{1}{\ln n}\right)^n$ converge or diverge?

5. Power series.

(a) (10 points) Find a power series representation for $\tan^{-1}(x)/x$.

(b) (10 points) Express $\int xe^x dx$ as a power series.

(c) (10 points) Find the interval of convergence of the power series $\sum_{n=0}^{\infty} \frac{3^n x^n}{n!}$.

6. Taylor series.

(a) (2 points) What is Taylor's formula?

(b) (10 points) Find the Taylor polynomial of order 3 generated by $f(x) = x \sin x$ at $x = \pi/2$.

(c) (10 points) Find the Maclaurin series generated by $f(x) = 2^x$.