

1.1 Solutions

$$\#2. \left[\begin{array}{ccc|c} 2 & 1 & -2 & 0 \\ 2 & -1 & -2 & 0 \\ 1 & 2 & -4 & 0 \end{array} \right] \begin{array}{l} R_1 - R_2 \\ R_1 - 2R_3 \end{array}$$

$$\rightarrow \left[\begin{array}{ccc|c} 2 & 1 & -2 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & -3 & 6 & 0 \end{array} \right] \begin{array}{l} 2R_1 - R_2 \\ 3R_2 + 2R_3 \end{array}$$

$$\rightarrow \left[\begin{array}{ccc|c} 4 & 0 & -4 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 12 & 0 \end{array} \right] 3R_1 + R_3$$

$$\rightarrow \left[\begin{array}{ccc|c} 12 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 12 & 0 \end{array} \right] \begin{array}{l} R_1/12 \\ R_2/2 \\ R_3/12 \end{array}$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right]$$

This gives the system

$$\begin{cases} x = 0 \\ y = 0 \\ z = 0 \end{cases}$$

which has solution

$$x=0, y=0, z=0.$$

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$$\left[\begin{array}{cc|c} 2 & 3 & 5 \\ 2 & 1 & 2 \\ -1 & -2 & 1 \end{array} \right] \begin{array}{l} R_1 - R_2 \\ R_1 - 2R_3 \end{array}$$

$$\rightarrow \left[\begin{array}{cc|c} 2 & 3 & 5 \\ 0 & 2 & 3 \\ 0 & 7 & 3 \end{array} \right] \begin{array}{l} 2R_1 - 3R_2 \\ 2R_3 - 7R_2 \end{array}$$

$$\rightarrow \left[\begin{array}{cc|c} 4 & \cancel{14} 0 & 1 \\ 0 & 2 & 3 \\ 0 & 0 & -15 \end{array} \right]$$

This augmented matrix corresponds to the system

$$\left\{ \begin{array}{l} 4x = 1 \\ 2y = 3 \\ 0 = -15 \end{array} \right.$$

which has no solution

\therefore the original system has no solution.

#21. By theorem 1.1 the system

$$9x - 2y + 17z = 0$$

$$13x + 81y - 27z = 0$$

has nontrivial solutions since it has
 $m=2$ equations, $n=3$ variables, and
 $m < n$.