1.3 Solutions

#3.

$$\left[\begin{array}{ccc|ccc} 1 & -2 & 3 & 1 & 0 & 0 \\ 2 & -1 & 4 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} -2R_1 + R_2 \\ R_1 - R_3 \end{array}$$

$$\rightarrow \left[\begin{array}{ccc|ccc} 1 & -2 & 3 & 1 & 0 & 0 \\ 0 & 3 & -2 & -2 & 1 & 0 \\ 0 & -3 & 2 & 1 & 0 & -1 \end{array}\right] R_2 + R_3$$

$$\rightarrow \left[\begin{array}{ccc|ccc} 1 & -2 & 3 & 1 & 0 & 0 \\ 0 & 3 & -2 & -2 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 & -1 \end{array}\right]$$

Reduced row echelon form is not I_3

$\therefore$ the matrix is not invertible.

$$\#5 \quad \left[\begin{array}{ccc|ccc} \multicolumn{3}{c|}{\overbrace{\hspace{4em}}^{A}} & & & \\ 0 & -2 & 1 & 1 & 0 & 0 \\ 2 & 4 & -1 & 0 & 1 & 0 \\ 2 & 1 & 2 & 0 & 0 & 1 \end{array}\right] \quad R_1 \leftrightarrow R_2$$

$$\rightarrow \left[\begin{array}{ccc|ccc} 2 & 4 & -1 & 0 & 1 & 0 \\ 0 & -2 & 1 & 1 & 0 & 0 \\ 2 & 1 & 2 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \\ \\ R_1 - R_3 \end{array}$$

$$\rightarrow \left[\begin{array}{ccc|ccc} 2 & 4 & -1 & 0 & 1 & 0 \\ 0 & -2 & 1 & 1 & 0 & 0 \\ 0 & 3 & -3 & 0 & 1 & -1 \end{array}\right] \begin{array}{l} 2R_2 + R_1 \\ \\ 3R_2 + 2R_3 \end{array}$$

$$\rightarrow \left[\begin{array}{ccc|ccc} 2 & 0 & 1 & 2 & 1 & 0 \\ 0 & -2 & 1 & 1 & 0 & 0 \\ 0 & 0 & -3 & 3 & 2 & -2 \end{array}\right] \begin{array}{l} 3R_1 + R_3 \\ 3R_2 + R_3 \\ \\ \end{array}$$

$$\rightarrow \left[\begin{array}{ccc|ccc} 6 & 0 & 0 & 9 & 5 & -2 \\ 0 & -6 & 0 & 6 & 2 & -2 \\ 0 & 0 & -3 & 3 & 2 & -2 \end{array}\right]$$

$$\rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3/2 & 5/6 & -1/3 \\ 0 & 1 & 0 & -1 & -1/3 & 1/3 \\ 0 & 0 & 1 & -1 & -2/3 & 2/3 \end{array}\right]$$

$$\therefore A^{-1} = \begin{bmatrix} 3/2 & 5/6 & -1/3 \\ -1 & -1/3 & 1/3 \\ -1 & -2/3 & 2/3 \end{bmatrix}$$

#11 $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

(a) $EA = \begin{bmatrix} 1 & 2 \\ 6 & 8 \end{bmatrix}$ is the matrix obtained from A by scaling row $i = 2$ by $t = 2$.

So E is the elementary diagonal matrix

$$E = D_2(2) = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

(b) $EA = \begin{bmatrix} 7 & 10 \\ 3 & 4 \end{bmatrix}$ is the matrix obtained from A row $i = 1$ by itself plus $t = 2$ times row $j = 2$ of A. So E is the unipotent matrix

$$E = U_{12}(2) = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

(c) $EA = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$ is the matrix obtained

from A by interchanging rows $i=1$ and $j=2$.

So E is the permutation matrix

$$E = P_{12} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$