1.6 Solutions

#4. $\begin{vmatrix} 2 & -1 & -3 \\ 1 & 1 & 3 \\ 6 & 0 & 0 \end{vmatrix} \leftarrow$

$= 6 \cdot \begin{vmatrix} -1 & -3 \\ 1 & 3 \end{vmatrix}$

$= 6(-3+3)$

$= 0$

$\therefore$ the matrix $\begin{bmatrix} 2 & -1 & -3 \\ 1 & 1 & 3 \\ 6 & 0 & 0 \end{bmatrix}$ is

not invertible.

#15. $A = \begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix}$ & $B = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix}$

(a) $\det A = \begin{vmatrix} 3 & -2 \\ 1 & 4 \end{vmatrix} = 12 + 2 = 14$

$\therefore$ A is invertible

$\det B = \begin{vmatrix} 1 & 2 \\ -2 & 3 \end{vmatrix} = 3 + 4 = 7$

$\therefore$ B invertible.

(b) $\det(AB) = \det(A) \cdot \det(B) = 14 \cdot 7 = 98$

$\det(A^{-1}) = \det(A)^{-1} = 14^{-1} = 1/14$

$$\det(B^T A^{-1}) = \det(B^T) \cdot \det(A^{-1})$$

$$= \det(B) \cdot \det(A)^{-1}$$

$$= 7 \cdot 1/14$$

$$= 1/2$$

(c) $A+B = \begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix}$

$= \begin{bmatrix} 4 & 0 \\ -1 & 7 \end{bmatrix}$

$\det(A+B) = 28 - 0 = 28$

on the other hand,

$\det(A) + \det(B) = 14 + 7 = 21$

$\therefore \det(A+B) = 28 \neq 21 = \det(A) + \det(B)$