2.1 Solutions

#6. Throughout let

$u = a+bi$, $v = c+di$, and $w = e+fi$

also let ~~c, d ∈ ℝ~~. $\alpha, \beta \in \mathbb{R}$.

$$
\begin{aligned}
(1)\quad u+v &= (a+bi)+(c+di) \\
&= (a+c)+(b+d)i \\
&= (c+a)+(d+b)i \\
&= (c+di)+(a+bi) \\
&= v+u
\end{aligned}
$$

$$
\begin{aligned}
(2)\quad u+(v+w) &= (a+bi)+\left[(c+di)+(e+fi)\right] \\
&= (a+bi)+\left[(c+e)+(d+f)i\right] \\
&= \left[a+(c+e)\right]+\left[b+(d+f)\right]i \\
&= \left[(a+c)+e\right]+\left[(b+d)+f\right]i
\end{aligned}
$$

$$= [(a+c)+(b+d)i] + (e+fi)$$

$$= [(a+bi) + (c+di)] + (e+fi)$$

$$= (u+v) + w$$

(3) The element $0 = 0 + 0i$ is the zero vector.

$$v + 0 = (c+di) + (0+0i)$$

$$= (c+0) + (d+0)i$$

$$= c + di$$

$$= v$$

(4) For each v define $-v = -c + (-d)i$ then

$$v + (-v) = (c+di) + (-c + (-d)i)$$

$$= (c-c) + (d-d)i$$

$$= 0 + 0i$$

$$= 0$$

(5) $\alpha(u+v) = \alpha\left[(a+bi)+(c+di)\right]$

$= \alpha\left[(a+c)+(b+d)i\right]$

$= \alpha(a+c) + \alpha(b+d)i$

$= (\alpha a+\alpha c) + (\alpha b+\alpha d)i$

$= (\alpha a + \alpha bi) + (\alpha c + \alpha di)$

$= \alpha(a+bi) + \alpha(c+di)$

$= \alpha u + \alpha v.$

(7) $\alpha(\beta v) = \alpha\left[\beta(c+di)\right]$

$= \alpha\left[\beta c + \beta d i\right]$

$= \alpha(\beta c) + \alpha(\beta d) i$

$= (\alpha\beta) c + (\alpha\beta) d i$

$= (\alpha\beta)\left[c + di\right]$

$= (\alpha\beta) v$

(8) $1 \cdot v = 1 \cdot (c+di)$

$= 1 \cdot c + 1 \cdot d i$

$= c + di$

$= v.$

#9. Let $u = z$, $v = z$, and $w = z$.

Let $c, d \in \mathbb{R}$.

(1) $u + v = z + z = v + u$

(2) $u + (v + w) = z + (z + z)$

$= z + z$

$= (z + z) + z$

$= (u + v) + w$

(3) Let $0 = z$, then

$v + 0 = z + z = z = v$.

(4) Define $-v = z$, then

$v + (-v) = z + z = z = 0$.

(5) $c(u+v) = c(z+z)$

$= cz$

$= z$

$= z + z$

$= cz + cz$

$= cu + cv$

(6) $(c+d)v = (c+d)z$

$= z$

$= z + z$

$= cz + dz$

$= cv + dv$

(7) $c(dv) = c(dz)$

$= c(z) = cz$

$= z$

$= (cd)z$

$= (cd)v$

(8) $1 \cdot v = 1 \cdot z = z = v.$