2.2 Solutions

#3. $V = F[a,b]$

(a) $W = \{f : f(a) = 0\}$ is a subspace.

(i) Let $f, g \in W$, then

$$(f+g)(a) = f(a) + g(a) = 0 + 0 = 0$$

$\therefore$ W is closed under addition

(ii) Let $f \in W$ and $c \in \mathbb{R}$, then

$$(cf)(a) = c \cdot f(a) = c \cdot 0 = 0.$$

$\therefore$ W is closed under scalar mult.

Hence, by (i) and (ii) W is a subspace.

(b) $W = \{f : f(a) = 1\}$ is not a subspace.

For example, $W$ is not closed under addition:

Let $f, g \in W$, then

$$(f+g)(a) = f(a) + g(a) = 1+1 = 2 \neq 1.$$

Alternatively, $W$ is not closed under scalar mult:

Let $f \in W$ and $c = 2$, then

$$(c \cdot f)(a) = (2f)(a) = 2 \cdot f(a) = 2 \cdot 1 = 2 \neq 1.$$

(a) $W = \left\{ f \in C[a,b] : \int_a^b f(x)\,dx = 0 \right\}$

is a subspace.

(i) Let $f, g \in W$, then $f+g$ is cts and integrable with

$$\int_a^b (f+g)(x)\,dx = \int_a^b f(x) + g(x)\,dx$$

$$= \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$$

$$= 0 + 0$$

$$= 0.$$

$\therefore$ W is closed under addition.

(ii) Let $f \in W$ and $c \in \mathbb{R}$, then $cf$ is cts. and integrable with

$$\int_a^b (cf)(x)\,dx = \int_a^b c \cdot f(x)\,dx = c \cdot \int_a^b f(x)\,dx$$

$$= c \cdot 0$$

$$= 0.$$

$\therefore$ W is closed under scalar mult.

Hence, by (i) & (ii) W is a subspace.

(a) $W = \{f \in D[a,b] : f' = f\}$ is a subspace.

(i) Let $f, g \in W$. Then $f+g$ is diff'ble and

$$(f+g)' = f' + g' = f + g$$

$\therefore$ $W$ is closed under addition.

(ii) Let $f \in W$ and $c \in \mathbb{R}$. Then $cf$ is differentiable and

$$(cf)' = c \cdot f' = c \cdot f$$

$\therefore$ $W$ is closed under scalar mult.

Hence, by (i) & (ii) $W$ is a subspace.

(f) $W = \{ f \in D[a,b] : f'(x) = e^x \}$ is

not a subspace.

For example $W$ is not closed under scalar

mult., say $c = 2$, then for $f \in W$

$$(cf)'(x) = (2f)'(x)$$

$$= 2 \cdot f'(x)$$

$$= 2 \cdot e^x$$

$$\neq e^x$$

#11.

$$c_1\begin{bmatrix}1\\-1\\0\end{bmatrix} + c_2\begin{bmatrix}1\\1\\1\end{bmatrix} + c_3\begin{bmatrix}2\\0\\1\end{bmatrix} = \begin{bmatrix}1\\-5\\-3\end{bmatrix}$$

$\rightsquigarrow$ this leads to the system of eqn's

$$\begin{aligned} c_1 + c_2 + 2c_3 &= 1 \\ -c_1 + c_2 \quad\quad &= -5 \\ c_2 + c_3 &= -3 \end{aligned}$$

$\rightsquigarrow$ this system has corresponding augmented matrix

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ -1 & 1 & 0 & -5 \\ 0 & 1 & 1 & -3 \end{array}\right] R_1 + R_2$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 2 & 2 & -4 \\ 0 & 1 & 1 & -3 \end{array}\right] R_2 - 2R_3$$

cont. $\longrightarrow$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 2 & 2 & -4 \\ 0 & 0 & 0 & 2 \end{array}\right]$$

the system has no solution

$\Rightarrow$ vector not in the span.