Solutions

#2.

(a) $v_1 = \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix}$, $v_2 = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$, $v_3 = \begin{bmatrix} 0 \\ -1 \\ 8 \end{bmatrix}$

check linear independence / dependence :

$$c_1 v_1 + c_2 v_2 + c_3 v_3 = \vec{0}$$

$$M = \left[\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 1 & -1 & -1 & 0 \\ 4 & 0 & 8 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$\Rightarrow$ trivial solution is the unique solution.

$\Rightarrow$ v_1, v_2, v_3 are L.I.

$\Rightarrow$ by theorem 2.12 v_1, v_2, v_3 are a basis of $\mathbb{R}^3$.

(b) $v_1 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$, $v_2 = \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}$, $v_3 = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$

Check linear independence / dependence :

$$c_1 v_1 + c_2 v_2 + c_3 v_3 = \vec{0}$$

$$\rightsquigarrow \left[\begin{array}{ccc|c} 3 & 4 & 3 & 0 \\ 2 & 4 & 1 & 0 \\ 1 & 0 & 2 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

system has infinitely many nontrivial solutions

$\Longrightarrow$ v_1, v_2, v_3 are L.D

$\Longrightarrow$ v_1, v_2, v_3 are not a basis

(c) These vectors are not a basis since $\dim(\mathbb{R}^3) = 3 \neq 4$.

(d) These vectors are not a basis since $\dim(\mathbb{R}^3) = 3 \neq 2$.

#12 $A = \begin{bmatrix} 1 & -1 & -1 & 2 & 0 \\ -2 & 1 & 1 & -1 & 0 \\ 1 & 1 & -2 & 1 & 1 \end{bmatrix}$

(a) $A \longrightarrow \begin{bmatrix} 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -4/3 & 1/3 \\ 0 & 0 & 1 & -5/3 & -1/3 \end{bmatrix}$

x_4, x_5 are free

$x_1 = x_4$

$x_2 = 4/3\, x_4 - 1/3\, x_5$

$x_3 = 5/3\, x_4 + 1/3\, x_5$

$\Longrightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_4 \begin{bmatrix} 1 \\ 4/3 \\ 5/3 \end{bmatrix} + x_5 \begin{bmatrix} 0 \\ -1/3 \\ 1/3 \end{bmatrix}$

basis for NS(A).

(b) The vectors $\begin{bmatrix} 1 & 0 & 0 & -1 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 1 & 0 & -4/3 & 1/3 \end{bmatrix}$,

and $\begin{bmatrix} 0 & 0 & 1 & -5/3 & -1/3 \end{bmatrix}$ is a basis

for $RS(A)$.

(c)

$$A^T = \begin{bmatrix} 1 & -2 & 1 \\ -1 & 1 & 1 \\ -1 & 1 & -2 \\ 2 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$RS(A^T)$ has basis $\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$,

and $\begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$,

$\Longrightarrow CS(A)$ has basis $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

(d) $\text{rank}(A) = 3$