Math 307 Practice Exam 1, Fall 2022

Name:

Question	Points	Score
1	16	
2	5	
3	5	
4	5	
5	8	
6	4	
7	6	
8	10	
9	7	
10	8	
Total:	74	

- You have 75 minutes to complete this exam.
- Please ask if anything seems confusing or ambiguous.
- You must show all your work, unless otherwise indicated. You will get almost no credit for solutions that are not fully justified.
- You may not use notes or calculators on this exam.

1. (16 points) True/False questions. No justification necessary.

(a) True False The zero space is a linear subspace of every vector space

(b) True False The only 1×1 matrices that are in row-reduced echelon form are [0] and [1].

(c) True False A matrix of size 3×4 can have 4 leading 1's.

(d) True False For all $n \times n$ matrices A, B and C, $A(B + C) = AB + AC$.

(e) True False If $\det(A) = 0$, then A is invertible.

(f) True False There exist nonzero square matrices A and B such that $(A+B)^2 = A^2 + B^2$.

(g) True False Multiplying a 3×3 matrix A on the left by

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

has the effect of scaling every entry in row 2 of A by 7.

(h) True False Every basis of $\mathbb{R}^9$ has exactly 9 vectors.

(i) True False For each invertible matrices A and B the matrix AB is invertible and $(AB)^{-1} = A^{-1}B^{-1}$.

(j) True False There is a 2×2 invertible matrix that has 3 entries that are 0.

(k) True False There is a 3×4 matrix A and a 4×3 matrix B such that $AB = BA$.

(l) True False A homogeneous system of linear equations $A\vec{x} = \vec{0}$ is always consistent.

(m) True False If A is not invertible, then A can be row reduced to a matrix with a row of zeros.

(n) True False For any matrix A, the matrix AA^T is symmetric.

(o) True False If $v_1, \ldots, v_n$ are linearly independent vectors in $\mathbb{R}^n$, then they form a basis for $\mathbb{R}^n$.

(p) True False For each $n \times n$ matrix A we have $\det(2A) = 2\det(A)$.

2. (5 points) Which of the following matrices are in RREF. No justification is necessary.

Matrix	Is in RREF	Is NOT in RREF
$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$		
$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$		
$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$		
$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$		
$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$		

3. (5 points) Which of the following are subspaces of the function space $F(\mathbb{R})$. No justification is necessary.

Set	Is a subspace	Is not a subspace
The space $C^\infty(\mathbb{R})$ of all smooth functions.		
The set of functions f in $F(\mathbb{R})$ such that $f(0) = 0$.		
The set of functions f in $F(\mathbb{R})$ such that $f(2) = 2$.		
The polynomials P_3.		
The set of all constant functions $f_c(x) = c$.		

4. (5 points) Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 8 & 2 \end{bmatrix}$$

5. (8 points) Define the matrices

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 0 \\ -1 & 0 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 1 & 3 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 \\ 1 & 2 \\ 1 & 5 \end{bmatrix}$$

Compute the following, or state that the expression is undefined.

(a) $A(-C)$

(b) CA

(c) $B^T B$

(d) BB^T

(e) C^3

6. (4 points) Solve the linear system of equations

$$\begin{aligned} x_1 - 7x_2 + x_5 &= 3 \\ x_3 - x_5 &= 2 \\ x_4 + x_5 &= 1 \end{aligned}$$

7. (6 points) Compute the determinant of the matrices:

(a) $A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 & 4 & 5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 3 & 4 & 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 & 3 & 4 & 5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 & 3 & 4 & 5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 2 & 3 & 4 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

(b) $B = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \end{bmatrix}$ *Hint*: do one row operation first.

(c) $C = \begin{bmatrix} 3 & 0 & 0 & 2 \\ 8 & 4 & 7 & 0 \\ 1 & 1 & 0 & -1 \\ 2 & 3 & 0 & 0 \end{bmatrix}$

8. (10 points) Let

$$\alpha = \left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 0 \\ 1 \end{bmatrix} \right\}$$

(a) Are the vectors in α linearly dependent or independent?

(b) Is the vector

$$\begin{bmatrix} 1 \\ -1 \\ 2 \\ 0 \end{bmatrix}$$

in the span of α.

(c) Do the vectors in α span $\mathbb{R}^4$? Why?

(d) Do the vectors in α form a basis for Span(α)? Why?

(e) Find $v \in \mathbb{R}^4$ if

$$[v]_\alpha = \begin{bmatrix} 2 \\ 7 \\ -1 \end{bmatrix}.$$

(more space for problem 8)

9. (7 points) Let

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}. \quad \vec{x} = \begin{bmatrix} x \\ y \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} s \\ t \end{bmatrix}$$

where the entries in A and $\vec{b}$ are real numbers and the entries in $\vec{x}$ are variables. Suppose A has nonzero determinant. Use Cramer's rule to derive formulas for the solutions to the system $A\vec{x} = \vec{b}$.

10. (8 points) Let

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 7 & 8 & 3 \\ 4 & 5 & 2 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 3 & 0 & 0 \\ 4 & 2 & 0 \\ 3 & 2 & 1 \end{bmatrix}.$$

Compute

(a) $\det(A^T B)$

(b) $\det((B^{-1})^3 A^2)$

(c) $\det(A - 2B)$