Solutions 5.1

#11. $T: M_{m \times n}(\mathbb{R}) \to M_{n \times m}(\mathbb{R})$

$$T(A) = A^T$$

linearity: $T(A+B) = (A+B)^T$

$= A^T + B^T$

$= T(A) + T(B)$

homogeneity: $T(c \cdot A) = (cA)^T$

$= c \cdot A^T$

$= c \cdot T(A)$

T <u>is</u> a linear transformation.

#12. $T: M_{n \times n}(\mathbb{R}) \to \mathbb{R}$, $T(A) = \det(A)$

Say for $n=3$.

$$\det(5 \cdot I) = 5^3 \cdot \det I = 5^3$$

$$5 \cdot \det(I) = 5$$

$$\therefore T(5I) \neq 5 \cdot T(I)$$

T is <u>not</u> linear.

#20. Find $T\begin{bmatrix}2\\1\\-4\end{bmatrix}$ if $T\begin{bmatrix}1\\-1\\0\end{bmatrix}=\begin{bmatrix}1\\0\\-1\\0\end{bmatrix}$

$T\begin{bmatrix}1\\0\\1\end{bmatrix}=\begin{bmatrix}2\\1\\0\\0\end{bmatrix}$, & $T\begin{bmatrix}0\\1\\-1\end{bmatrix}=\begin{bmatrix}1\\0\\0\\-1\end{bmatrix}$

Find c_1, c_2, c_3 s.t.

$$c_1\begin{bmatrix}1\\-1\\0\end{bmatrix}+c_2\begin{bmatrix}1\\0\\1\end{bmatrix}+c_3\begin{bmatrix}0\\1\\-1\end{bmatrix}=\begin{bmatrix}2\\1\\-4\end{bmatrix}$$

$$\rightsquigarrow \left[\begin{array}{ccc|c}1&1&0&2\\-1&0&1&1\\0&1&-1&-4\end{array}\right] \xrightarrow{RREF} \left[\begin{array}{ccc|c}1&0&0&5/2\\0&1&0&-1/2\\0&0&1&7/2\end{array}\right]$$

$$\Rightarrow T\begin{bmatrix}2\\1\\-4\end{bmatrix}=5/2\,T\begin{bmatrix}1\\-1\\0\end{bmatrix}-1/2\,T\begin{bmatrix}1\\0\\1\end{bmatrix}+7/2\,T\begin{bmatrix}0\\1\\-1\end{bmatrix}$$

$$=5/2\begin{bmatrix}1\\0\\-1\\0\end{bmatrix}-1/2\begin{bmatrix}2\\1\\0\\0\end{bmatrix}+7/2\begin{bmatrix}1\\0\\0\\-1\end{bmatrix}$$

$$=\begin{bmatrix}10/2\\-1/2\\-5/2\\-7/2\end{bmatrix}$$