5.2 Solutions

#4: $$(S-4T)\begin{bmatrix} x \\ y \end{bmatrix} = S\begin{bmatrix} x \\ y \end{bmatrix} - 4T\begin{bmatrix} x \\ y \end{bmatrix}$$

$$= \begin{bmatrix} 2x-y \\ x+2y \end{bmatrix} - 4\begin{bmatrix} x+3y \\ x-y \end{bmatrix}$$

$$= \begin{bmatrix} 2x-y-4(x+3y) \\ x+2y-4(x-y) \end{bmatrix}$$

$$= \begin{bmatrix} -2x-13\cancel{x}y \\ -3x+6y \end{bmatrix}$$

#6 $$TS\begin{bmatrix} x \\ y \end{bmatrix} = T\begin{bmatrix} 2x-y \\ x+2y \end{bmatrix}$$

$$= \begin{bmatrix} (2x-y)+3(x+2y) \\ (2x-y)-(\cancel{2}x+2y) \end{bmatrix}$$

$$= \begin{bmatrix} 5x-5y \\ x-3y \end{bmatrix}$$

#12 Find a basis for Kernel of D^2+2D+2

$$p(\lambda) = \lambda^2 + 2\lambda + 2$$

find roots:

$$\lambda = \frac{-2 \pm \sqrt{4-4(1)(2)}}{2} = -1 \pm i$$

pick $r = -1+i \Rightarrow a = -1$ & $b = 1$

<u>Claim:</u> $y_1 = e^{-x}\cos x$ and $y_2 = e^{-x}\sin x$

Since the dimension of the kernel is 2, it suffices to show y_1 & y_2 are L.I.

Compute the Wronskian

$$W(y_1, y_2) = \begin{vmatrix} e^{-x}\cos x & e^{-x}\sin x \\ -e^{-x}\cos x - e^{-x}\sin x & -e^{-x}\sin x + e^{-x}\cos x \end{vmatrix}$$

$$= e^{-x}\begin{vmatrix} \cos x & \sin x \\ -\cos x - \sin x & \cos x - \sin x \end{vmatrix}$$

$$= e^{-x}\left(\cos^2 x - \cos x \sin x + \cos x \sin x + \sin^2 x\right)$$

$$= e^{-x}$$

Since $W(y_1, y_2) = e^{-x}$ is never zero

on $(-\infty, \infty)$, the vectors y_1 & y_2 are

L.I.