## 5.5

#6. $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$. from 5.4 (see solutions)

$$\dim E_{\frac{5+\sqrt{33}}{2}} + \dim E_{\frac{5-\sqrt{33}}{2}} = 1+1 = 2$$

$\therefore$ A is diagonalizable.

$$D = P^{-1}AP, \text{ where}$$

$$D = \begin{bmatrix} \frac{5+\sqrt{33}}{2} & 0 \\ 0 & \frac{5-\sqrt{33}}{2} \end{bmatrix} \quad \text{and} \quad P = \underbrace{\begin{bmatrix} \sqrt{33}-3 & \sqrt{33}+3 \\ 6 & -6 \end{bmatrix}}_{\text{may vary by a scalar.}}$$

#12. $A = \begin{bmatrix} 1 & 2 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & 0 \end{bmatrix}$

$A$ has eigenvalues $\lambda = 1$ (mult 3).

$E_1$ has basis $\left\{ \begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix} \right\}$

So $\dim E_1 = 1 \neq 3$

so $A$ is not diagonalisable.

#18. $A = \begin{bmatrix} 3 & 1 & -1 \\ 0 & 0 & -2 \\ 0 & 1 & 2 \end{bmatrix}$ by 5.4

$\dim E_3 + \dim E_{1+i} + \dim E_{1-i} = 1+1+1 = 3$

$\therefore$ A is diagonalizable.

$D = P^{-1} A P$ where

$D = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1+i & 0 \\ 0 & 0 & 1-i \end{bmatrix}$ and $P = \begin{bmatrix} 1 & 1 & 1 \\ 0 & -1+i & -1-i \\ 0 & 1 & 1 \end{bmatrix}$