6.1

#1. Show that $Y = \begin{bmatrix} c_1 e^{2x} + c_2 e^{3x} \\ 2c_1 e^{2x} + c_2 e^{3x} \end{bmatrix}$ is

a solution to $Y' = \begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix} Y$

LHS: $$Y' = \begin{bmatrix} 2c_1 e^{2x} + 3c_2 e^{3x} \\ 4c_1 e^{2x} + 3c_2 e^{3x} \end{bmatrix}$$

RHS: $$\begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix} Y = \begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} c_1 e^{2x} + c_2 e^{3x} \\ 2c_1 e^{2x} + c_2 e^{3x} \end{bmatrix}$$

$$= \begin{bmatrix} 4(c_1 e^{2x} + c_2 e^{3x}) - 1(2c_1 e^{2x} + c_2 e^{3x}) \\ 2(c_1 e^{2x} + c_2 e^{3x}) + 1(2c_1 e^{2x} + c_2 e^{3x}) \end{bmatrix}$$

$$= \begin{bmatrix} 2c_1 e^{2x} + 3c_2 e^{3x} \\ 4c_1 e^{2x} + 3c_2 e^{3x} \end{bmatrix}$$

#3 $\quad Y_1 = \begin{bmatrix} e^{2x}\cos 3x \\ -e^{2x}\sin 3x \end{bmatrix}, \quad Y_2 = \begin{bmatrix} e^{2x}\sin 3x \\ e^{2x}\cos 3x \end{bmatrix}$

$$w\big(Y_1(x), Y_2(x)\big) = \begin{vmatrix} e^{2x}\cos 3x & e^{2x}\sin 3x \\ -e^{2x}\sin 3x & e^{2x}\cos 3x \end{vmatrix}$$

$$= \left(e^{2x}\right)^2 \begin{vmatrix} \cos 3x & \sin 3x \\ -\sin 3x & \cos 3x \end{vmatrix}$$

$$= e^{4x}\left(\cos^2 3x + \sin^2 3x\right)$$

$$= e^{4x}$$

$$\neq 0$$

$\therefore$ Y_1 & Y_2 are L.I. in $C^{\infty}(\mathbb{R})^2$

#7 The general solution to

$$Y' = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 4 \end{bmatrix} Y$$

is $$Y_H = \begin{bmatrix} c_1 e^{-x} \\ c_2 \\ c_3 e^{4x} \end{bmatrix}$$