Math 244 Final, Spring 2022

Name:

Question	Points	Score
1	10	
2	10	
3	14	
4	9	
5	10	
6	11	
7	18	
8	12	
9	15	
10	0	
Total:	109	

- You have 2 hours to complete this exam.
- All work must be your own.
- You may use one page of notes that you may turn in for extra credit.
- You must show all your work. You will get almost no credit for solutions that are not fully justified.
- Answer the questions in the space provided. Use the back of the page for scratch work or if you require additional space for your answers. Clearly indicate what is a solution, and what is scratch work.
- No electronic devices are authorized with the exception of a scientific calculator.
- Good luck!

1. (10 points) Find the volume of the tetrahedron in the first octant bounded by the coordinate planes and the plane $x + y + z = 1$.

2. (10 points) Use Stokes' theorem to compute

$$\iint_S \text{curl } \mathbf{F} \cdot d\mathbf{S},$$

where $\mathbf{F} = \langle y, z, x \rangle$ and S is the outward oriented paraboloid $z = 1 - x^2 - y^2$ that sits above the xy-plane.

3. (14 points) Let C be the solid cone bounded by the surfaces $z = \sqrt{x^2 + y^2}$ and $z = 4$. Use the divergence theorem to compute the outward flux of $\mathbf{F} = \langle x^3, y^3, xy^2 \rangle$ across the boundary of C.

4. (9 points) Set up the iterated integral that finds the surface area of the helicoid parametrized by

$$\mathbf{r}(r, \theta) = \langle r \cos\theta, r \sin\theta, \theta \rangle,$$

where $0 \leq r \leq 1$ and $0 \leq \theta \leq 2\pi$.

5. (10 points) Evaluate the line integral

$$\int_C y^2 dx + x^2 dy,$$

where C is the circle $x^2 + y^2 = 4$ oriented counterclockwise.

6. (11 points) Compute the triple integral

$$\iiint_E \frac{dxdydz}{(x^2+y^2+z^2)^{3/2}},$$

where E is the solid bounded between the spheres $x^2+y^2+z^2=1$ and $x^2+y^2+z^2=16$.

7. Let $\mathbf{F} = 2xze^y\mathbf{i} + x^2ze^y\mathbf{j} + x^2e^y\mathbf{k}$.

   (a) (5 points) Show that $\mathbf{F}$ is conservative.

   (b) (10 points) Find a potential function for $\mathbf{F}$.

   (c) (3 points) Find the work done by $\mathbf{F}$ on a particle moving along any piecewise smooth path beggining and ending on the x-axis.

8. (12 points) Find the center of mass of the thin lamina L bounded by the curves $y = 1$ and $y = x^2$ with density $\rho(x, y) = y$.

9. (15 points) Compute the flux of the vector field $\mathbf{F} = yz\mathbf{i} + zx\mathbf{j} + xy\mathbf{k}$ across the portion of the plane $x + z = 5$ oriented upward parametrized by

$$\mathbf{r}(x, y) = x\mathbf{i} + y\mathbf{j} + (5 - x)\mathbf{k},$$

where $x^2 + y^2 \leq 1$.

10. (10 points (bonus)) Let $\mathbf{x} = \langle x, y, z \rangle$ be the position vector. Compute the outward flux of the vector field

$$\mathbf{F} = \frac{\mathbf{x}}{|\mathbf{x}|^3},$$

across the ellipsoid

$$\frac{x^2}{9} + \frac{y^2}{4} + \frac{z^2}{6} = 1.$$