

# Math 244 Exam 3, Spring 2023

Name:

Question	Points	Score
1	6	
2	7	
3	8	
4	10	
5	8	
6	0	
Total:	39	

- You have 50 minutes to complete this exam.
- Please ask if anything seems confusing or ambiguous.
- You must show all your work unless the problem states otherwise. You will get almost no credit for solutions that are not fully justified.
- You may use a 3x5 notecard with notes.
- No electronic devices are authorized with the exception of a scientific calculator.
- You do not need to simplify your answers.
- The back side of each page can be used as scratch paper.
- Good luck!

Homework	
Exam 1	
Exam 2	
Exam 3	
Total	

1. (6 points) The vector field  $\mathbf{F} = 2x \sin(y)\mathbf{i} + x^2 \cos(y)\mathbf{j}$  is conservative, find a potential function for  $\mathbf{F}$ .

2. (7 points) Evaluate

$$\int_C \nabla(\arctan(xyz)) \cdot d\mathbf{r}$$

where  $C$  is the path of straight line segments from  $(1, 1, 1)$  to  $(1, 1, 0)$  to  $(1, 0, 0)$  to  $(0, 0, 0)$ .

3. (8 points) Evaluate

$$\oint_C (x^2y^3 + 2y)dx + x^3y^2dy$$

where  $C$  is the boundary of the rectangle  $[-1, 1] \times [-1, 1]$  oriented counterclockwise.

4. Let  $\mathbf{F}$  be the vector field  $\mathbf{F} = \langle y \cos z, x \sin z, xy \sin(z^2) \rangle$ .
- (a) (4 points) Determine if  $\mathbf{F}$  is conservative.
  - (b) (6 points) Find the work done by  $\mathbf{F}$  on a particle moving along the path  $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle$ ,  $-1 \leq t \leq 1$ .

5. Let  $C$  be the semicircular arc of  $y^2 + z^2 = 1$  that lies above the  $xy$ -plane.
- (a) (3 points) Find a parametrization of  $C$ .
  - (b) (5 points) Evaluate  $\int_C e^y dy$ .
  - (c) (1 point (bonus)) Sketch what the integral in part (b) represents.

6. (5 points (bonus)) Let  $\mathbf{F}$  be a vector field. A vector field  $\mathbf{G}$  is a *vector potential* for  $\mathbf{F}$  if  $\text{curl}(\mathbf{G}) = \mathbf{F}$ . Find vector potentials for  $\mathbf{F}_1 = \langle 2, 3, 4 \rangle$  and  $\mathbf{F}_2 = \langle x, 0, 0 \rangle$ .