Math 244 Final, Spring 2023

Name:

Question	Points	Score
1	27	
2	33	
3	20	
4	21	
5	22	
6	20	
Total:	143	

- You have 120 minutes to complete this exam.
- Please ask if anything seems confusing or ambiguous.
- You must show all your work unless the problem states otherwise. You will get almost no credit for solutions that are not fully justified.
- You may use a 5x8 notecard with notes. If you make use of any derivations we made in lecture that you have written on your notecard, clearly make a reference to your notecard. Otherwise you will lose points for not showing work.
- No electronic devices are authorized with the exception of a scientific calculator.
- You do not need to simplify your answers.
- The back side of each page can be used as scratch paper.
- Good luck!

1. Let

$$\mathbf{F} = \langle y - z, z - x, x + y \rangle,$$

and let S be the top half of the unit sphere oriented upward.

(a) (3 points) Find curl($\mathbf{F}$). Is $\mathbf{F}$ conservative?

(b) (14 points) Evaluate the surface integral $\iint_S \operatorname{curl}(\mathbf{F}) \cdot d\mathbf{S}$ directly by using the parametrization

$$\mathbf{r}(\phi, \theta) = \langle \cos\theta \sin\phi, \sin\theta \sin\phi, \cos\phi \rangle, \quad 0 \le \phi \le \frac{\pi}{2}, \quad 0 \le \theta \le 2\pi.$$

(c) (10 points) Verify your answer to part (b) with Stokes' theorem.

(more space)

2. Let

$$\mathbf{F} = \langle x - y, y - x, z \rangle,$$

and let E be the solid bounded by the paraboloid $z = 1 - x^2 - y^2$ and the plane $z = 0$. Orient the boundary ∂E outward.

(a) (20 points) Evaluate the surface integral $\iint_{\partial E} \mathbf{F} \cdot d\mathbf{S}$ directly. *Hint*: the surface has two pieces.

(b) (13 points) Verify your answer from part (a) with the divergence theorem.

(more space)

3. (20 points) Let S be the portion of the helicoid parametrized by

$$\mathbf{r}(r,\theta) = r\cos\theta\mathbf{i} + r\sin\theta\mathbf{j} + r\mathbf{k}, \quad 0 \le r \le 1, \quad 0 \le \theta \le 2\pi.$$

(a) Find the surface area of S.

(b) Evaluate the surface integral $\iint_S x \, dS$.

4. Let

$$\mathbf{F} = y^2 \sin(z)\mathbf{i} + 2xy\sin(z)\mathbf{j} + (xy^2\cos(z) + 1)\mathbf{k},$$

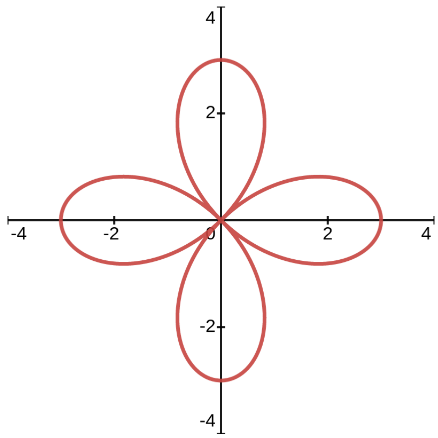
and let C be the top half of the unit circle $y^2 + z^2 = 1$.

(a) (12 points) Find a potential function for $\mathbf{F}$.

(b) (9 points) Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$.

5. (22 points) Find the center of mass of the snow cone bounded by the sphere $x^2+y^2+z^2 = 4$ and the cone $z = \sqrt{x^2+y^2}$ with density $\rho(x, y, z) = x^2 + y^2 + z^2$.

6. Consider the rose curve $r = 3\cos(2\theta)$ pictured below, and let C be one traversal of the rose (parametrized using the angle θ).



(a) (9 points) Find the area enclosed by the rose curve.

(b) (11 points) Evaluate the line integral $\int_C (x^3 - 7y)dx + (\arctan(y^3) + 9x)dy$.