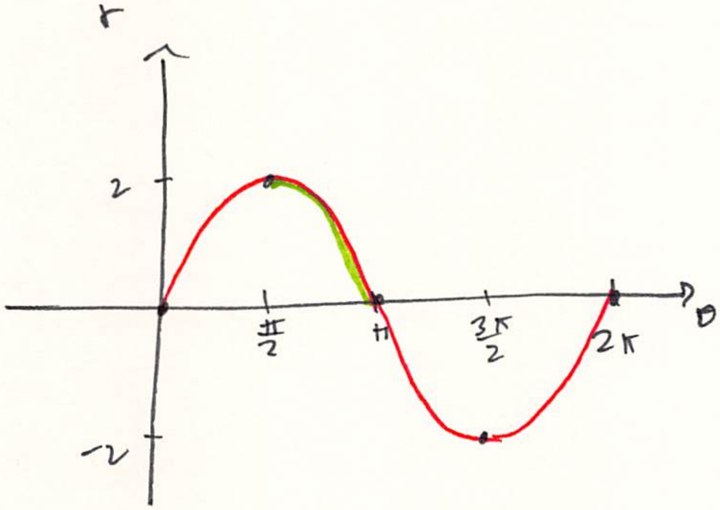
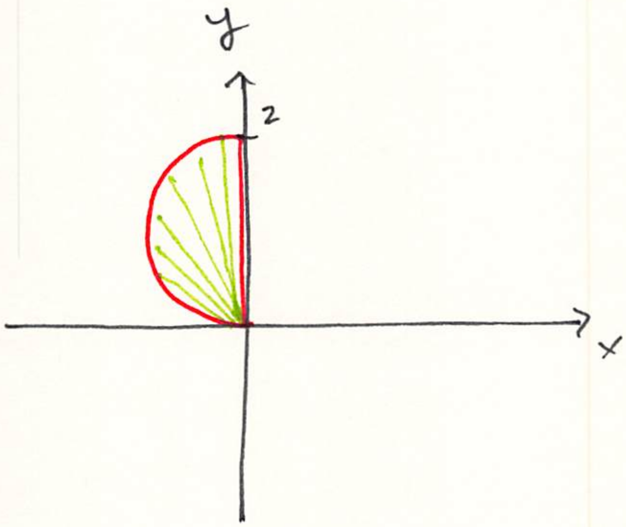
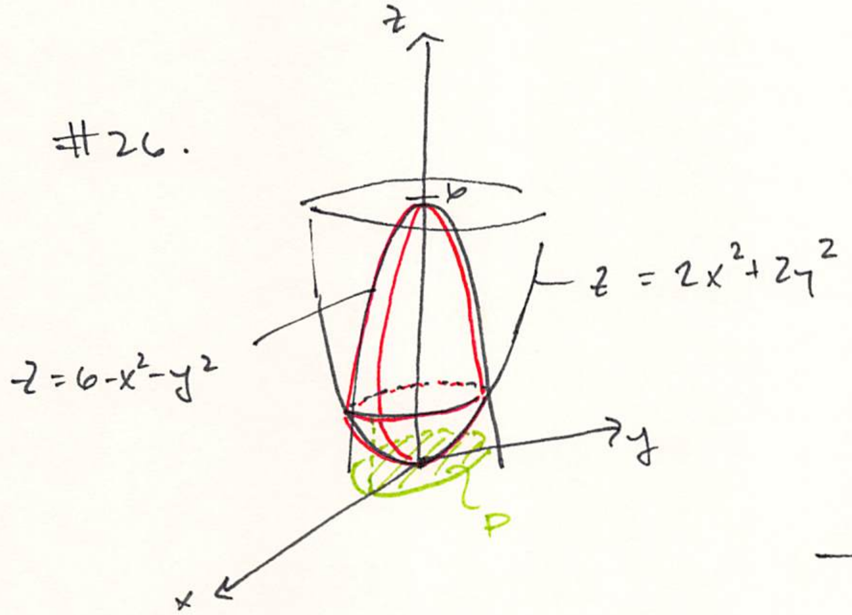
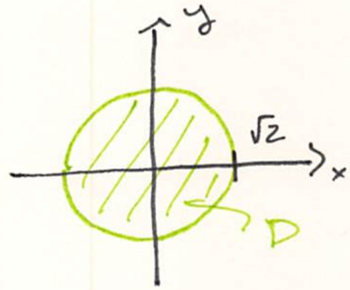
#6. $$\int_{\pi/2}^{\pi}\int_{0}^{2\sin\theta} r\,dr\,d\theta$$





#26.





$$6-x^2-y^2 = 2x^2+2y^2$$
$$x^2+y^2 = 2$$
$$r = \sqrt{2}$$

D in polar

$0 \le \theta \le 2\pi$

$0 \le r \le \sqrt{2}$

$$V = \iint_D 6-x^2-y^2-(2x^2+2y^2)\, dA$$
$$= \iint_D 6-3x^2-3y^2\, dA$$
$$= \int_0^{2\pi}\int_0^{\sqrt{2}} (6-3r^2)\, r\, dr\, d\theta$$
$$= 2\pi \cdot \left[6r - r^3\right]_0^{\sqrt{2}}$$
$$= 2\pi\left[6\sqrt{2} - 2\sqrt{2}\right] = 8\pi\sqrt{2}$$

#32. $\displaystyle\int_0^2 \int_0^{\sqrt{2x-x^2}} \sqrt{x^2+y^2}\; dy\, dx$

$y = \sqrt{2x - x^2}$ $\xrightarrow[\text{square}]{\text{complete}}$ $y = \sqrt{1-(x-1)^2}$

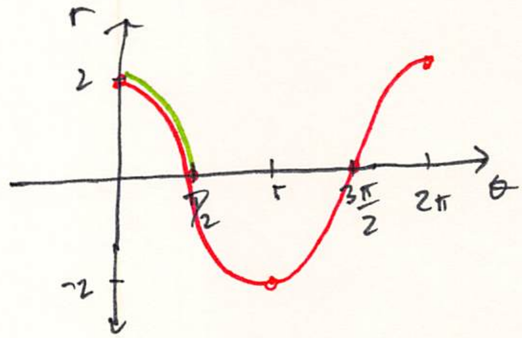
top half of unit circle shifted right 1 unit.

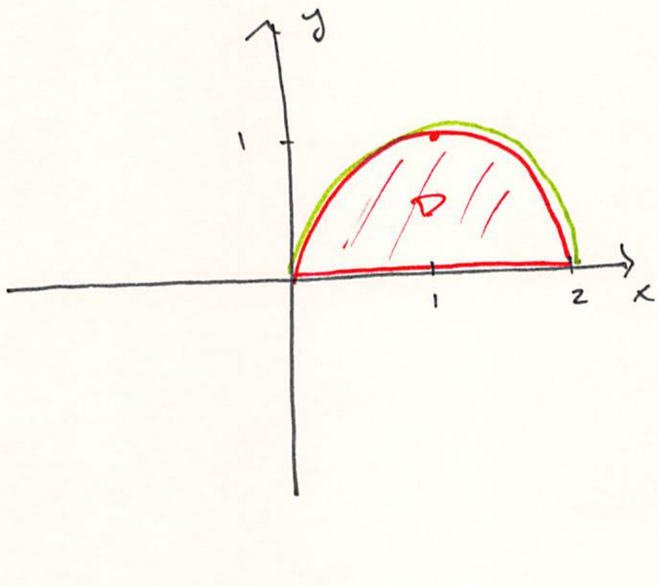
$y^2 = 2x - x^2$

$\Rightarrow x^2 + y^2 = 2x$

$\Rightarrow r^2 = 2r\cos\theta$

$\Rightarrow r = 2\cos\theta$





D in polar:

$0 \le \theta \le \pi/2$

$0 \le r \le 2\cos\theta$

$$\int_0^2 \int_0^{\sqrt{2x-x^2}} \sqrt{x^2+y^2}\, dy\, dx$$

$$= \int_0^{\pi/2} \int_0^{2\cos\theta} \sqrt{r^2} \cdot r\, dr\, d\theta$$

$$= \int_0^{\pi/2} \frac{r^3}{3} \Big|_0^{2\cos\theta} d\theta$$

$$= \frac{8}{3} \int_0^{\pi/2} \cos^3\theta\, d\theta = \frac{8}{3} \int_0^{\pi/2} (1-\sin^2\theta) \cos\theta\, d\theta$$

$u = \sin\theta \qquad u(0) = 0$

$du = \cos\theta\, d\theta \qquad u(\pi/2) = 1$

$$= \frac{8}{3} \int_0^1 1 - u^2\, du$$

$$= \frac{8}{3} \left(u - u^3/3 \right)_0^1$$

$$= \frac{8}{3} \left(1 - 1/3 \right) = \frac{16}{9}$$