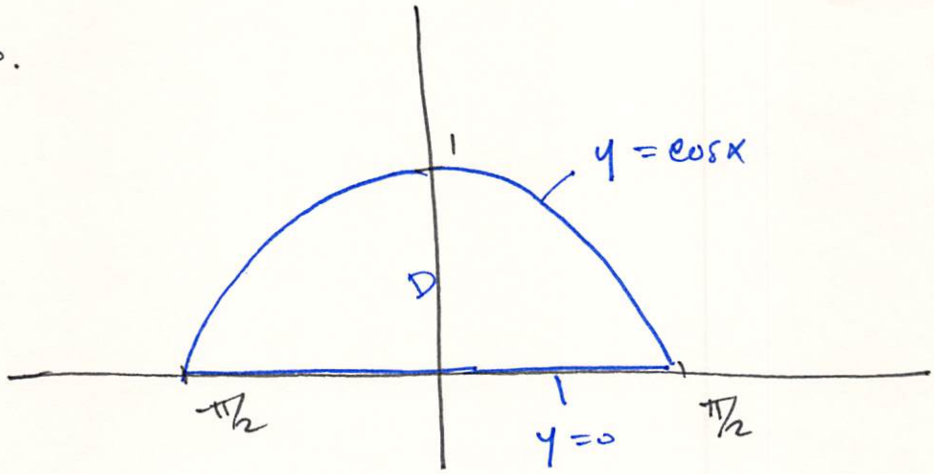
#10.



$\rho(x,y) = y \implies \rho(-x,y) = y = \rho(x,y)$

$\therefore$ by symmetry $\bar{x} = 0$.

$$m = \iint_D \rho(x,y)\, dA = \int_{-\pi/2}^{\pi/2} \int_0^{\cos x} y \, dy\, dx$$

$$= \int_{-\pi/2}^{\pi/2} \frac{y^2}{2} \Big|_0^{\cos x} dx$$

$$= \frac{1}{2} \int_{-\pi/2}^{\pi/2} \cos^2 x \, dx$$

even

$$= \frac{1}{2} \cdot 2 \cdot \int_0^{\pi/2} \cos^2 x \, dx$$

$$= \frac{1}{2}\int_0^{\pi/2} 1 + \cos 2x \, dx$$

$$= \frac{1}{2}\left[x + \frac{1}{2}\cancel{\sin} 2x \right]_0^{\pi/2}$$

$$= \pi/4$$

$$M_x = \iint_D y \cdot \rho(x,y)\, dA$$

$$= \int_{-\pi/2}^{\pi/2} \int_0^{\cos x} y^2 \, dy \, dx$$

$$= \int_{-\pi/2}^{\pi/2} \frac{y^3}{3} \Big|_0^{\cos x} dx$$

$$= \frac{1}{3}\int_{-\pi/2}^{\pi/2} \cos^3 x \, dx = 6 \int_0^{\pi/2} \cos^3 x \, dx$$

even

$$= 6 \int_0^{\pi/2} (1 - \sin^2 x) \cos x \, dx$$

$$= 6 \int_0^1 1 - u^2 \, du$$

$$= 6 \left[u - u^3/3 \right]_0^1$$

$$= 6 \left[1 - 1/3 \right]$$

$$= {}^{2}\cancel{6} \cdot \frac{2}{\cancel{3}}$$

$$= 4$$

$$\therefore \bar{y} = \frac{M_x}{m} = \frac{4}{\pi/4} = \frac{16}{\pi}$$

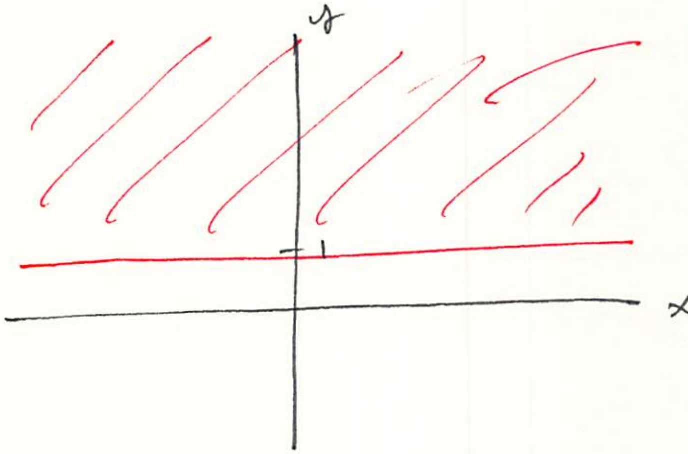
$$= \frac{1}{10}\left[\lim_{b\to\infty}\left(-5\cancelto{0}{e^{-b/5}} + 5e^{-1/5}\right)\right]\cdot\left[\lim_{a\to\infty}\left(-2\cancelto{0}{e^{-a/2}} + 2\right)\right]$$

$$= \frac{1}{10}\left(5e^{-1/5}\right)\cdot 2$$

$$= e^{-1/5}$$

#29.

(bi)



D:
$-\infty < x < \infty$
$1 \le y < \infty$

$$P(Y \ge 1) = \iint_D f(x,y)\, dA$$

$$= \int_1^\infty \int_0^\infty \frac{1}{10} e^{-x/2 - y/5}\, dx\, dy$$

$$= \frac{1}{10} \int_1^\infty e^{-y/5}\, dy \cdot \int_0^\infty e^{-x/2}\, dx$$

$$= \frac{1}{10} \cdot \left[\lim_{b\to\infty} \int_1^b e^{-y/5}\, dy \right] \cdot \left[\lim_{a\to\infty} \int_0^a e^{-x/2}\, dx \right]$$

$$= \frac{1}{10} \left[\lim_{b\to\infty} \left(-5e^{-y/5} \right)_1^b \right] \cdot \left[\lim_{a\to\infty} \left(-2e^{-x/2} \right)_0^a \right]$$

$\longrightarrow$