#12. $x^2+y^2+z^2=4z$

complete the square on z.

$\to x^2+y^2+(z-2)^2=4$

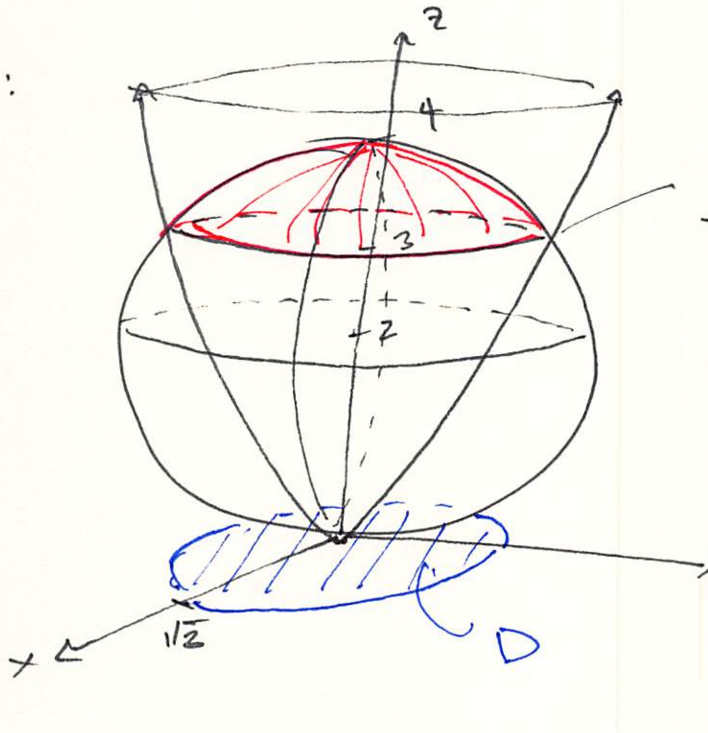
Find where paraboloid $z=x^2+y^2$ intersects the sphere $\underbrace{x^2+y^2}_{z}+z^2=4z$

$\Rightarrow z+z^2=4z$

$\Rightarrow z^2-3z=0$

$z=0,3$

Sketch:



$z=3$

$\Rightarrow x^2+y^2=3$

radius $\sqrt{3}$

function: $z = \sqrt{4-x^2-y^2} + 2$

$$z_x = \frac{1}{2}(4-x^2-y^2)^{-1/2} \cdot (-2x)$$

$$\Longrightarrow \quad z_x^2 = \frac{x^2}{4-x^2-y^2}$$

similarly $\quad z_y^2 = \dfrac{y^2}{4-x^2-y^2}$

$$SA = \iint_D \sqrt{1 + \frac{x^2}{4-x^2-y^2} + \frac{y^2}{4-x^2-y^2}} \, dA$$

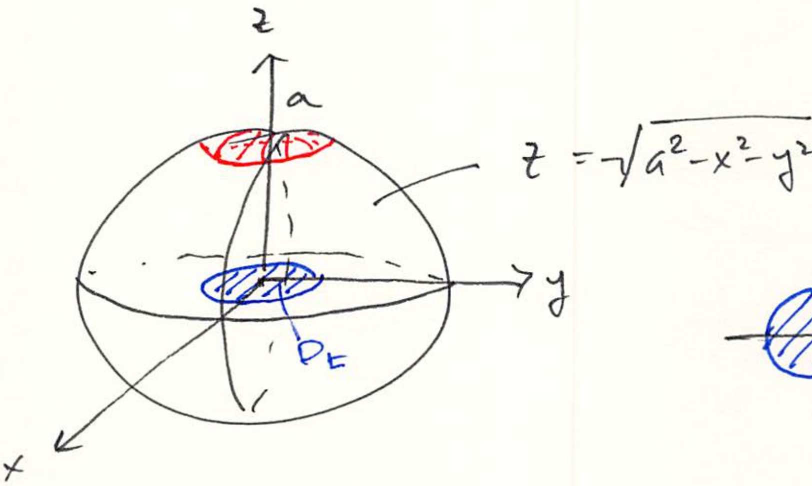
$$= \int_0^{2\pi} \int_0^{\sqrt{3}} \sqrt{\frac{4}{4-r^2}} \; r \, dr \, d\theta \qquad u = 4-r^2$$

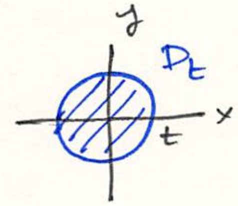
$$= 2\pi \cdot 2 \cdot \int_4^1 \frac{1}{\sqrt{u}} \cdot \left(-\frac{1}{2} dr\right)$$

$$= 4\pi \cdot \sqrt{u} \Big|_1^4$$

$$= 4\pi(2-1) = 4\pi$$

#22.





$$z_x^2 = \frac{x^2}{a^2-x^2-y^2} \quad , \quad z_y^2 = \frac{y^2}{4-x^2-y^2}$$

$$\text{Surface Area} = 2 \cdot \text{x Surface Area of top half}$$

$$= 2 \cdot \lim_{t \to a} \iint_{D_t} \sqrt{1 + z_x^2 + z_y^2} \, dA$$

$$= 2 \cdot \lim_{t \to a} \iint_{D_t} \sqrt{1 + \frac{x^2}{a^2-x^2-y^2} + \frac{y^2}{a^2-x^2-y^2}} \, dA$$

$$= 2 \cdot \lim_{t \to a} \int_0^{2\pi} \int_0^t \frac{a}{\sqrt{a^2-r^2}} \, r \, dr \, d\theta$$

$$u = a^2 - r^2$$

$$du = -2r \, dr$$

$$= 2 \cdot \lim_{t \to a} \left[2\pi a \int_{a^2}^{a^2 - t^2} \frac{1}{\sqrt{u}} \cdot \left(-\frac{1}{2} du\right) \right]$$

$$= 4\pi a \cdot \lim_{t \to a} \sqrt{u} \Big|_{a^2 - t^2}^{a^2}$$

$$= 4\pi a \cdot \lim_{t \to a} \left(a - \sqrt{a^2 - t^2} \right)$$

$$= 4\pi a (a - 0)$$

$$= 4\pi a^2$$