#8. $\displaystyle\int_0^1\int_0^1\int_0^{2-x^2-y^2} xye^z \, dz\, dy\, dx$

$$= \int_0^1\int_0^1 xy\left[e^z\right]_0^{2-x^2-y^2} dy\, dx$$

$$= \int_0^1\int_0^1 xy\left[e^{2-x^x-y^2} - 1\right] dy\, dx$$

$$u = 2-x^2-y^2 \qquad u(0) = 2-x^2$$

$$du = -2y\,dy \qquad u(1) = 1-x^2$$

$$= \int_0^1\int_{2-x^2}^{1-x^2} x\left[e^u - 1\right]\left(-\tfrac{1}{2}\,du\right) dx$$

$$= \frac{1}{2}\int_0^1 x\left[-e^u + u\right]_{2-x^2}^{1-x^2} dx$$

$$= \frac{1}{2}\int_0^1 x\left[-(1-x^2) + (2-x^2) - e^{1-x^2} + e^{2-x^2}\right] dx$$

$$= \frac{1}{2}\int_0^1 x\,dx - \frac{1}{2}\int_0^1 x e^{1-x^2}\,dx + \frac{1}{2}\int_0^1 x e^{2-x^2}\,dx$$

$u = 1-x^2$

$du = -2x\,dx$

$u(0) = 1$

$u(1) = 0$

$u = 2-x^2$

$du = -2x\,dx$

$u(0) = 2$

$u(1) = 1$

$$= \frac{1}{2}\,\frac{x^2}{2}\Big|_0^1 + \frac{1}{4}\int_1^0 e^u\,du + \frac{1}{4}\int_1^2 e^u\,du$$

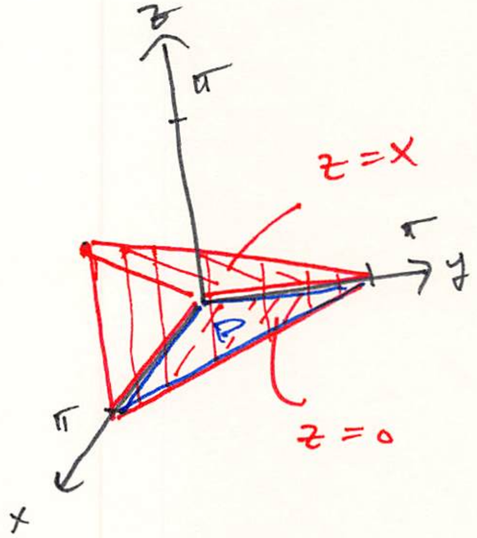
$$= \frac{1}{4} + \frac{1}{4}(1-e) + \frac{1}{4}(e^2-e)$$

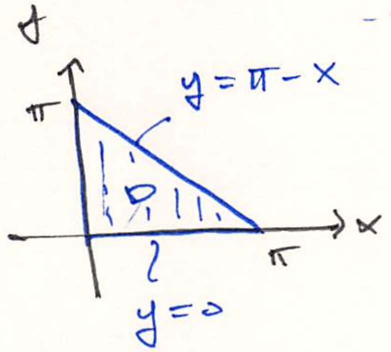
$$= \frac{1}{4}(1+1-e+e^2-e)$$

$$= \frac{1}{4}(1-2e+e^2)$$

$$= \frac{1}{4}(1-e)^2$$

#12. $\iiint_E \sin y \, dV$





$$= \int_0^{\pi} \int_0^{\pi - x} \int_0^{x} \sin y \, dz\, dy\, dx$$

$$= \int_0^{\pi} \int_0^{\pi - x} \sin y \cdot x \, dy\, dx$$

$$= \int_0^{\pi} x \left[-\cos y \right]_0^{\pi - x} dx$$

$$= \int_0^{\pi} x \left[1 - \cos(\pi - x) \right] dx$$

area of triangle

$$= \underbrace{\int_0^{\pi} x\,dx}_{} - \int_0^{\pi} x\cos(\pi - x)\,dx$$

IBP

x		$\cos(\pi-x)$
	$+$	$\sin(\pi-x)$
1		
	$-$	$-\cos(\pi-x)$
0		

$$= \frac{1}{2}\cdot\pi\cdot\pi - \left[x\cancel{\sin(\pi-x)} + \cos(\pi-x)\right]_0^{\pi}$$

$$= \frac{\pi^2}{2} - \left(1-(-1)\right)$$

$$= \frac{\pi^2}{2} - 2$$