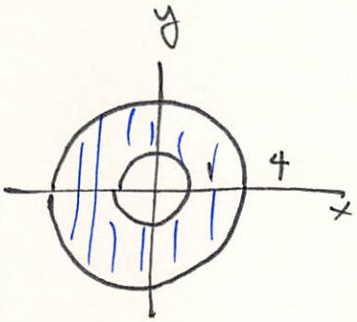
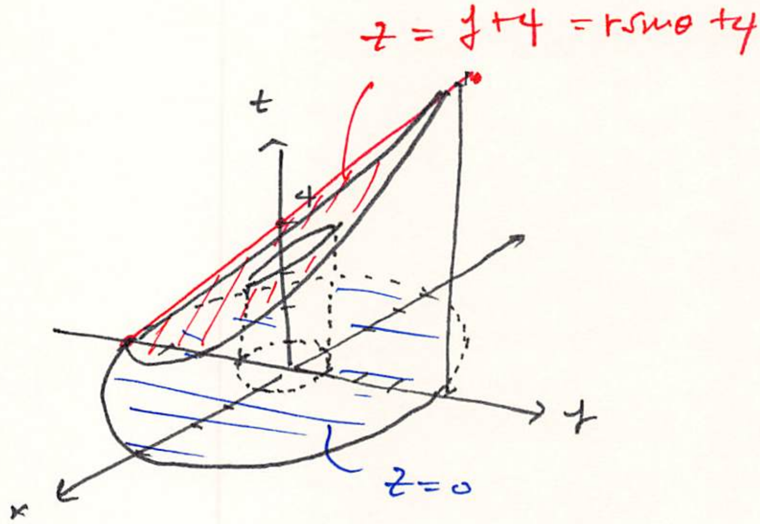
15.8

#20. $\iiint_E (x-y)\, dV$





$$= \int_0^{2\pi}\int_1^4\int_0^{r\sin\theta+4} (r\cos\theta - r\sin\theta)\, r \, dz\, dr\, d\theta$$

$$= \int_0^{2\pi}\int_1^4 r^2(\cos\theta - \sin\theta)\, z \Big|_0^{r\sin\theta+4} dr\, d\theta$$

$$= \int_0^{2\pi}\int_1^4 r^2(\cos\theta - \sin\theta)(r\sin\theta + 4)\, dr\, d\theta$$

$$= \int_0^{2\pi}\int_1^4 r^2\left(r\sin\theta\cos\theta - r\sin^2\theta + \cancel{4\cos\theta} - \cancel{4\sin\theta}\right) dr\, d\theta$$

$$= \cancel{\int_0^0\int_1^4 r^3 u\, du} - \int_0^{2\pi} \frac{1-\cos 2\theta}{2} \cdot \frac{r^4}{4}\Big|_1^4 d\theta$$

$$= -\frac{1}{8}\left[\theta - \frac{1}{2}\cancel{\sin 2\theta}\right]_0^{2\pi} \cdot (4^4 - 1)$$

$$= -\frac{1}{8}\cdot 2\pi(255) = -\frac{255}{4}\pi$$

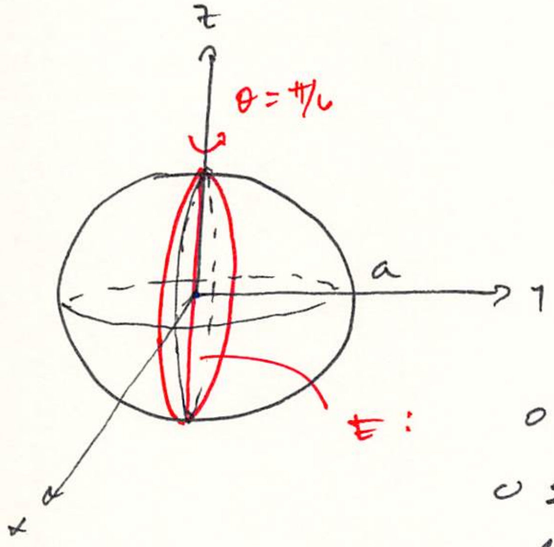
15.8

#20.

$$\int_{\pi/2}^{2\pi}\int_{0}^{\pi/2}\int_{1}^{2} f(x,y,z)\ \rho^2 \sin\phi \ d\rho\, d\phi\, d\theta$$

$$= \int_{\pi/2}^{2\pi}\int_{0}^{\pi/2}\int_{1}^{2} f(\rho\sin\phi\cos\theta,\ \rho\sin\phi\sin\theta,\ \rho\cos\phi)\ \rho^2 \sin\phi \ d\rho\, d\phi\, d\theta$$

#36.



$0 \le \rho \le a$

$0 \le \theta \le \pi/6$

$0 \le \phi \le \pi$

$$V(E) = \iiint_E 1 \, dV$$

$$= \int_0^{\pi/6} \int_0^{\pi} \int_0^{a} 1 \cdot \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

$$= \pi/6 \cdot \int_0^{\pi} \frac{\rho^3}{3} \Big|_0^a \sin\phi \, d\phi$$

$$= \frac{a^3 \pi}{18} \cdot \Big[-\cos\phi \Big]_0^{\pi}$$

$$= \frac{a^3 \pi}{9}$$