#26. $$\iint_R \sin(9x^2 + 4y^2)\, dA$$

$$R : \quad 9x^2 + 4y^2 \le 1$$

$$= \iint_R \sin\big((\underbrace{3x}_{u})^2 + (\underbrace{2y}_{v})^2 \big)\, dA$$

$$T^{-1} : \quad u = 3x \quad \& \quad v = 2y$$

$$\Rightarrow \quad T : \quad x = u/3 \quad \& \quad y = v/2$$

$$J(T) = \begin{vmatrix} 1/3 & 0 \\ 0 & 1/2 \end{vmatrix} = 1/6$$

$$T^{-1}(R) : \quad 9x^2 + 4y^2 \le 1$$

$$\Leftrightarrow \quad (3x)^2 + (2y)^2 \le 1$$

$$\Leftrightarrow \quad u^2 + v^2 \le 1$$

then

$$\iint_R \sin(9x^2+4y^2)\,dA$$

$$= \iint_{T^{-1}(R)} \sin(u^2+v^2)\cdot\left|\tfrac{1}{6}\right|\,du\,dv$$

$$= \frac{1}{6}\int_0^{2\pi}\int_0^1 \sin(r^2)\, r\,dr\,d\theta$$

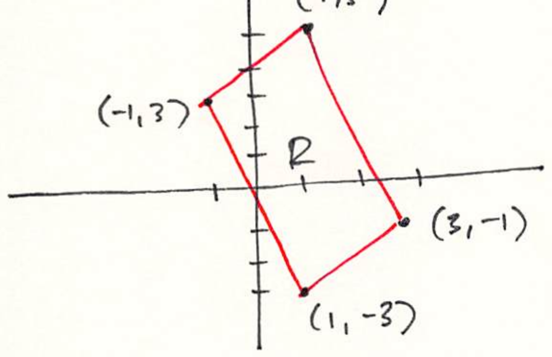
$$= \frac{\pi}{3}\cdot\int_0^1 \sin u\ \left(\tfrac{1}{2}\,du\right)$$

$$= \frac{\pi}{6}\left(-\cos u\right)\Big|_0^1$$

$$= \frac{\pi}{6}\left(1-\cos(1)\right)$$

#16. $\displaystyle\iint_R (4x+8y)\,dA$

R :



$T: \quad x = \frac{1}{4}(u+v), \quad y = \frac{1}{4}(v-3u)$

$$J(T) = \begin{vmatrix} 1/4 & 1/4 \\ -3/4 & 1/4 \end{vmatrix} = 1/16 + 3/16 = 1/4$$

T is linear so we just need to find at where the corners of R map to under T^{-1}.

$$\begin{array}{rl} u+v &= 4x \\ -(-3u+v &= 4y) \\ \hline 4u &= 4x-4y \end{array}$$

$$\Rightarrow \quad u = x - y.$$

$$\Longrightarrow \quad u + v = 4x$$

$$(x-y) + v = 4x$$

$$v = 3x + y$$

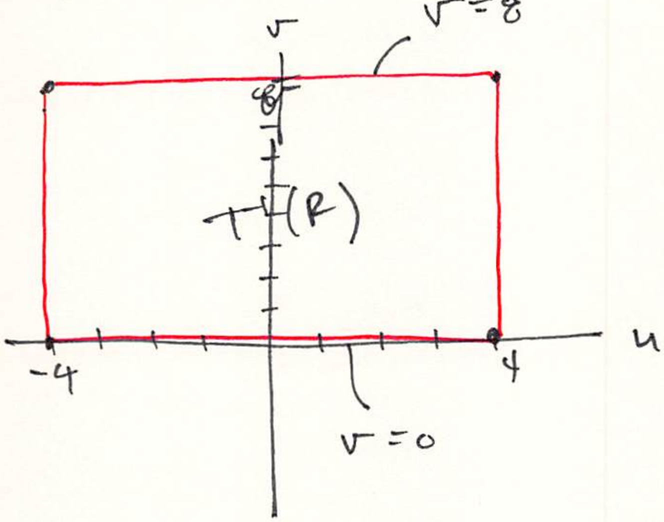
$$T(x,y) = (x-y,\ 3x+y)$$

$$T(1,5) = (-4,8)$$

$$T(-1,3) = (-4,0)$$

$$T(1,-3) = (4,0)$$

$$T(3,-1) = (4,8)$$



$$\iint_R (4x+8y)\,dA$$

Jacobian 1/4

$$= \int_{-4}^{4}\int_{0}^{8} 4\left(\tfrac{1}{4}(u+v)\right) + 8\left(\tfrac{1}{4}(v-3u)\right) dv\,du$$

$$= \frac{1}{4}\int_{-4}^{4}\int_{0}^{8} u+v+2v-6u \; dv\,du$$

$$= \frac{1}{4}\int_{-4}^{4}\int_{0}^{8} 3v-5u \; dv\,du$$

$$= \frac{1}{4}\int_{-4}^{4} \frac{3}{2}v^2\Big|_0^8 - 5uv\Big|_0^8 \; du$$

$$= \frac{1}{4}\int_{-4}^{4} 96-40u \; du$$

$$= 24u - 5u^2 \Big|_{-4}^{4}$$

$$= (96-80) - (-96-80)$$

$$= 192$$