

No. 2

$$\# 20. \vec{F} = \langle x+y^2, xz, y+z \rangle$$

$$\vec{F}(t) = \langle t^2, t^3, -2t \rangle, \quad 0 \leq t \leq 2$$

$$\vec{F}(\vec{F}(t)) = \langle t^2 + (t^3)^2, t^2 \cdot (-2t), t^3 - 2t \rangle$$

$$= \langle t^2 + t^6, -2t^3, t^3 - 2t \rangle$$

$$\vec{F}'(t) = \langle 2t, 3t^2, -2 \rangle$$

$$\vec{F}(\vec{F}(t)) \cdot \vec{F}'(t)$$

$$= (t^2 + t^6)(2t) + (-2t^3)(3t^2) + (t^3 - 2t)(-2)$$

$$= \cancel{2t^3} + 2t^7 - 6t^5 - \cancel{2t^3} + 4t$$

$$= 2t^7 - 6t^5 + 4t$$

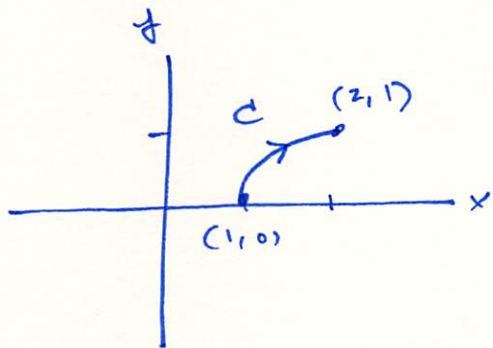
$$\int_C \vec{F} \cdot d\vec{r} = \int_0^2 2t^7 - 6t^5 + 4t \, dt$$

$$= \left[\frac{t^8}{4} - t^6 + 2t^2 \right]_0^2$$

$$= \frac{2^8}{4} - 2^6 + 2 \cdot 2^2$$

$$= 8$$

#40. $\vec{F} = \langle x^2, ye^x \rangle$



$$\vec{r}(t) = (t^2+1)\vec{i} + t\vec{j}$$

$$= \langle t^2+1, t \rangle$$

$$0 \leq t \leq 1$$

$$\vec{F} = \langle (t^2+1)^2, t \cdot e^{t^2+1} \rangle$$

$$\vec{F}' = \langle 2t, 1 \rangle$$

$$\vec{F} \cdot \vec{F}' = (t^2+1)^2 \cdot 2t + t e^{t^2+1}$$

$$\text{Work} = \int_0^1 \vec{F} \cdot d\vec{r}$$

$$= \int_0^1 (t^2+1)^2 2t + t e^{t^2+1} dt$$

$$= \int_0^1 (t^4 + 2t^2 + 1) 2t dt + \int_0^1 t e^{t^2+1} dt$$

$$= 2 \int_0^1 t^5 + 2t^3 + t \, dt + \int_0^1 t e^{t^2+1} \, dt$$

$$u = t^2 + 1$$
$$du = 2t \, dt$$

$$u(0) = 1$$

$$u(1) = 2$$

$$= 2 \left[\frac{t^6}{6} + \frac{t^4}{2} + \frac{t^2}{2} \right]_0^1 + \int_1^2 e^u \left(\frac{1}{2} du \right)$$

$$= 2 \left[\frac{1}{3} + \frac{1}{2} + \frac{1}{2} \right] + \frac{1}{2} e^u \Big|_1^2$$

$$= \frac{7}{3} + \frac{1}{2} (e^2 - e)$$