16.3

#15. $\vec{F} = \langle \underset{f_x}{\sin y},\ \underset{f_y}{x\cos y + \cos z},\ \underset{f_z}{-y\sin z} \rangle$

$C: \vec{r}(t) = \langle \sin t,\ t,\ 2t \rangle, \quad 0 \le t \le \pi/2$

$$f = \int f_x \, dx = \int \sin y \, dx = x\sin y + g(y,z)$$

$$\Rightarrow f_y = x\cos y + \frac{\partial g}{\partial y} = x\cos y + \cos z$$

$$\Rightarrow \frac{\partial g}{\partial y} = \cos z$$

$$\Rightarrow g(y,z) = \int \cos z \, dy = y\cos z + h(z)$$

$$f = x\sin y + y\cos z + h(z)$$

$$\Rightarrow f_z = 0 + y(-\sin z) + h'(z) = -y\sin t$$

$$h'(z) = 0$$

$$\rightarrow h(z) = c$$

$$\therefore f(x,y,z) = x\sin y + y\cos z + c$$

(b) $\int_C \vec{F} \cdot d\vec{r} = \int_C \nabla(x\sin y + y\cos z + c) \cdot d\vec{r}$

$= \Big[x\sin y + y\cos z + c \Big]_{\vec{r}(0)}^{\vec{r}(\pi/2)}$

$= \Big[x\sin y + y\cos z + c \Big]_{(0,0,0)}^{(1,\, \pi/2,\, \pi)}$

$= \left(1 \cdot 1 + \frac{\pi}{2} \cdot (-1) \right) - \left(0 \cdot 0 + 0 \cdot 1 \right)$

$= 1 - \frac{\pi}{2}$

#20. $\displaystyle\int_C \overbrace{\sin y}^{P}\, dx + (\overbrace{x\cos y - \sin y}^{Q})\, dy$

C: any path from $(2,0)$ to $(1,\pi)$.

$$\text{curl}\langle P, Q, 0\rangle = (Q_x - P_y)\vec{k}$$
$$= (\cos y - \cos y)\vec{k}$$
$$= \vec{0}$$

$\Rightarrow$ $\vec{F} = \langle P, Q\rangle$ is conservative

$\Rightarrow$ $\displaystyle\int_C \vec{F}\cdot d\vec{r}$ is path independent.

(1) $f_x = P$ and $f_y = Q$ (2)

$$f = \int f_x\, dx \overset{①}{=} \int P\, dx = \int \sin y\, dx$$
$$= x\sin y + g(y)$$

$$\Rightarrow f_y = x\cos y + g'(y) \overset{②}{=} x\cos y - \sin y$$

$$\Rightarrow \quad g'(y) = -\sin y$$

$$\Rightarrow \quad g(y) = \int -\sin y \, dy$$

$$= \cos y + c$$

$$\therefore \; f(x,y) = x\sin y + \cos y + c$$

$$\int_C \vec{F}\cdot d\vec{r} = \int_C \nabla\left(x\sin y + \cos y + c\right)\cdot d\vec{r}$$

$$= \Big[x\sin y + \cos y + c\Big]_{(2,0)}^{(1,\pi)}$$

$$= \Big(1\cdot 0 + (-1)\Big) - \left(2\cdot 0 + 1\right)$$

$$= -2$$

16.5

#18 $\vec{F} = \langle e^x \sin yz,\ z e^x \cos yz,\ y e^x \cos yz \rangle$

$$\operatorname{curl} \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ D_x & D_y & D_z \\ e^x \sin yz & z e^x \cos yz & y e^x \cos yz \end{vmatrix}$$

$$= \vec{i}\left(e^x \cos(yz) - e^x \cos(yz)\right) - \vec{j}\left(y e^x \cos yz - y e^x \cos yz\right)$$
$$+ \vec{k}\left(z e^x \cos(yz) - z e^x \cos(yz)\right)$$

$$= \vec{0}$$

$\vec{F}$ is conservative.