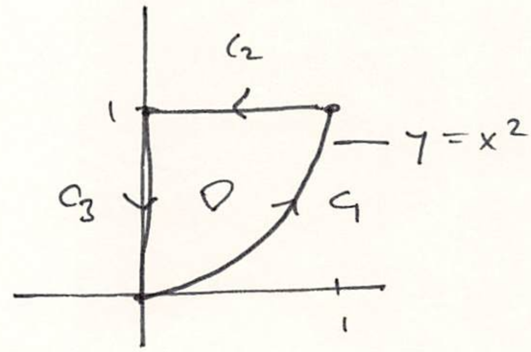
16.4

#4. $\oint_C x^2y^2\,dx + xy\,dy$



$C_1 : \vec{r}(t) = \langle t, t^2 \rangle, \quad 0 \le t \le 1$

$\vec{r}\,'(t) = \langle 1, 2t \rangle$

$$\int_{C_1} x^2y^2\,dx + xy\,dy = \int_0^1 t^2 \cdot (t^2)^2 \cdot 1 + t \cdot (t^2)(2t)\,dt$$

$$= \int_0^1 t^6 + 2t^4\,dt$$

$$= t^7/7 + 2t^5/5 \Big]_0^1$$

$$= 1/7 + 2/5 = 19/35$$

$-C_2 : \vec{r}(t) = \langle t, 1 \rangle, \quad 0 \le t \le 1$

$\vec{r}\,'(t) = \langle 1, 0 \rangle$

$$\int_{C_2} x^2y^2\,dx + xy\,dy = -\int_0^1 (t)^2(1)^2 \cdot 1 + (t)(1) \cdot 0\,dt$$

$$= -t^3/3 \Big]_0^1 = -1/3$$

$-C_3: \quad \vec{r}(t) = \langle 0, t \rangle, \qquad 0 \le t \le 1$

$$\int_{C_3} x^2y^2\,dx + xy\,dy = -\int_0^1 (0)^2(t)^2 \cdot 0 + (0)(t)\cdot 1 \; dt$$

$$= 0.$$

$$\therefore \oint_C x^2y^2\,dx + xy\,dy = \tfrac{19}{35} + \left(-\tfrac{1}{3}\right) + 0$$

$$= \tfrac{22}{105}$$

$$\oint_C x^2y^2\,dx + xy\,dy = \iint_D \frac{\partial}{\partial x}[xy] - \frac{\partial}{\partial y}[x^2y^2] \; dA$$

$$= \iint_D y - 2x^2y \; dA$$

$$= \int_0^1 \int_{x^2}^1 y - 2x^2y \; dy\,dx$$

$$= \int_0^1 (1-2x^2)\left[\frac{y^2}{2}\right]_{x^2}^{1} dx$$

$$= \int_0^1 (1-2x^2)\left[\frac{1}{2} - \frac{x^4}{2}\right] dx$$

$$= \frac{1}{2}\int_0^1 1 - x^4 - 2x^2 + 2x^6 \, dx$$

$$= \frac{1}{2}\left[x - \frac{x^5}{5} - \frac{2x^3}{3} + \frac{2x^7}{7}\right]_0^1$$

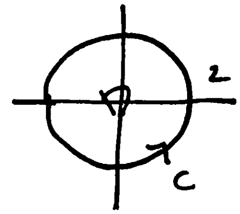
$$= \frac{1}{2}\left[1 - \frac{1}{5} - \frac{2}{3} + \frac{2}{7}\right]$$

$$= \frac{1}{2}\left[\frac{105 - 21 - 70 + 30}{105}\right]$$

$$= \frac{22}{105}$$

16.4

#9. $\displaystyle\int_C y^3 dx - x^3 dy$



$$= \iint_D \frac{\partial}{\partial x}\left[-x^3\right] - \frac{\partial}{\partial y}\left[y^3\right] dA$$

$$= \iint_D -3x^2 - 3y^2 \, dA$$

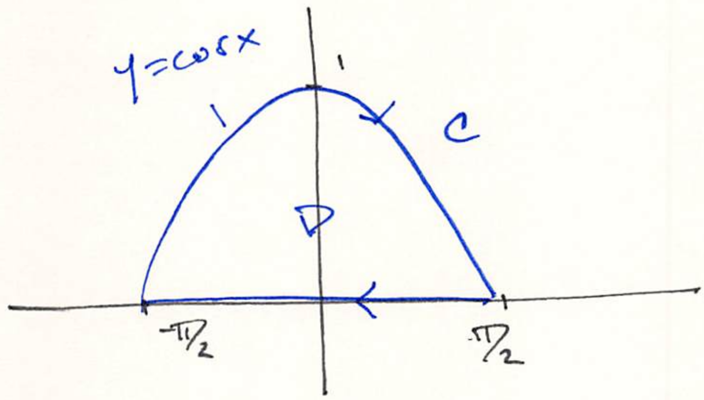
$$= -3 \cdot \int_0^{2\pi}\int_0^2 r^2 \cdot r \, dr\, d\theta$$

$$= -6\pi \left[r^4/4 \right]_0^2$$

$$= -6\pi \cdot 4$$

$$= -24\pi$$

#12. $\vec{F} = \langle \overbrace{e^{-x} + y^2}^{P}, \overbrace{e^{-y} + x^2}^{Q} \rangle$



$$\int_{-C} \vec{F} \cdot d\vec{r} = \iint_D Q_x - P_y \, dA$$

$$= \int_{-\pi/2}^{\pi/2} \int_0^{\cos x} 2x - 2y \; dy \, dx$$

$$= \int_{-\pi/2}^{\pi/2} 2xy - y^2 \Big|_0^{\cos x} dx$$

$$= \int_{-\pi/2}^{\pi/2} \underbrace{2x\cos x}_{\text{odd}} - \underbrace{\cos^2 x}_{\text{even}} \; dx$$

$$= -2 \int_0^{\pi/2} \frac{1 + \cos 2x}{2}\, dx$$

$$= -\left[x + \frac{1}{2} \cancel{\sin 2x} \right]_0^{\pi/2}$$

$$= -\pi/2$$

Hence,

$$\int_C \vec{F} \cdot d\vec{r} = -\int_{-C} \vec{F} \cdot d\vec{r}$$

$$= -\left(-\pi/2\right)$$

$$= \pi/2$$