16.5

#6.

(a) $\operatorname{curl} \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ D_x & D_y & D_z \\ \tan^{-1}(xy) & \tan^{-1}(yz) & \tan^{-1}(zx) \end{vmatrix}$

$$= \vec{i}\left(0 - \frac{y}{(yz)^2+1}\right) - \vec{j}\left(\frac{z}{(zx)^2+1} - 0\right) + \vec{k}\left(0 - \frac{x}{(xy)^2+1}\right)$$

$$= -\frac{y}{(yz)^2+1}\vec{i} - \frac{z}{(zx)^2+1}\vec{j} - \frac{x}{(xy)^2+1}\vec{k}$$

(b) $\operatorname{div} \vec{F} = \frac{y}{(xy)^2+1} + \frac{z}{(yz)^2+1} + \frac{x}{(zx)^2+1}$

v.f - vector field

s.f - scalar field

#12.

(a) curl f — nonsense

(b) grad f — v.f.

(c) div $\vec{F}$ — s.f.

(d) curl (grad f) — v.f.

(e) grad $\vec{F}$ — nonsense

(f) grad (div $\vec{F}$) — v.f.

(g) div (grad f) — s.f.

(h) grad (div f) — nonsense

(i) curl (curl $\vec{F}$) — v.f.

(j) div (div $\vec{F}$) — nonsense

(k) grad f × div $\vec{F}$ — nonsense

(l) div (curl (grad f)) — s.f.

#26. $\vec{F} = \langle P, Q, R \rangle$

$$\text{curl}(f\vec{F}) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ D_x & D_y & D_z \\ fP & fQ & fR \end{vmatrix}$$

$$= \vec{i}\left[f_y R + fR_y - (f_z Q + fQ_z)\right]$$

$$- \vec{j}\left[f_x R + fR_x - (f_z P + fP_z)\right]$$

$$+ \vec{k}\left[f_x Q + fQ_x - (f_y P + fP_y)\right]$$

$$= (fR_y - fQ_z)\vec{i} - \vec{j}(fR_x - fP_z) + (fQ_x - fP_y)\vec{k}$$

$$+ \vec{i}(f_y R - f_z Q) - \vec{j}(f_x R - f_z P) + \vec{k}(f_x Q - f_y P)$$

$$= f\left[\vec{i}\left(R_y - Q_z\right) - \vec{j}\left(R_x - P_z\right) + \vec{k}\left(Q_x - P_y\right)\right]$$

$$+ \vec{i}\left(f_y R - f_z Q\right) - \vec{j}\left(f_x R - f_z P\right) + \vec{k}\left(f_x Q - f_y P\right)$$

$$= f \cdot \operatorname{curl}(F) + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ f_x & f_y & f_z \\ P & Q & R \end{vmatrix}$$

$$= f \cdot \operatorname{curl}(\vec{F}) + (\nabla f) \times \vec{F}$$